MODERN MANAGEMENT REVIEW 2013
MMR, vol. XVIII, 20 (3/2013), pp. 159-172 July-September

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AN EXAMPLE OF OPTIMIZING THE SIZE OF THE QUEUE IN THE NONDETERMINISTIC LOGISTIC SYSTEMS

The work presents the example of nondeterministic model of the operating system and optimizing the size of the queue in such a system. As far as modeling is concerned, Markov models [3][4][13][16][17] have been used. The work also presents a method of determining crucial work parameters of the nondeterministic system.

The first and the second chapter contain some general information connected with modeling, as well as equations from which nondeterministic models of the systems with a countable number of conditions have been constructed. They are also shown the distinctions of discrete models and the ones continuing in time. The third chapter provides the information on geometric distributions. The distribution itself is the model element being presented in the other part of the work. The fourth chapter presents the definitions of essential operating parameters and their implementation. It is particularly important to highlight two major parameters: the average time for operating one notification, as well as the average time of notification in a system. The theorem of the notification time in a system has also been proved. The fifth chapter deals with the sample model of nondeterministic operating system. In such a system operating is shown in a pipelining way. Between two operating nodes in a system there is a queue, the size of which determines the efficiency of a system. The sixth chapter presents the optimal size of a queue for a previously mentioned model. For this purpose the relationship based on the cost of manufacturing system, depending on the size of the queue and the price of a system in its efficiency was adopted.

Keywords: optimizing, size of the queue, nondeterministic logistic systems, Markov models.

1. INTRODUCTION

As far as modeling and system optimization in general technical problems are concerned, three different issues can be distinguished:
1) modeling
2) determining the parameters of the system
3) optimization

The aim of modeling is to provide the actual system in mathematical form (systems of linear equations, differential equations, graphs, etc.)

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Models can be used to simulate the behavior of the system in unusual conditions, as conducting experiments on real systems sometimes is not possible or simply too expensive. In the queuing theory models are generally presented in the form of linear equations of ordinary differential equations (continuous models) or linear difference equations (discrete models). In the given conditions these models are simplified to the linear equations of the systems. For the queuing theory, it is characteristic that in the systems of modeling probabilities of system's existence in provided states are their variables.

The aim of determining the parameters of the system is to obtain information about the system. Sometimes some interesting parameters are included in the model itself. However, they are often hidden and it is necessary to determine them.

In the queuing theory such parameters are, for example: the average time of the node engaging, the average number of notifications in the queues or the average time of the notification's staying in a system.

Optimization aims to find the best solution (control or the structure) in the set of many available solutions.

To talk about the best control or structure it is necessary to define the function of objective. The objective function is the measure of the quality control or the structure of the system. Its arguments are the parameters of the system which were mentioned above. The objective function is the balance of profit and loss connected with the decision about a control or a structure of a system. The objective function very often has an economic interpretation.

In the queuing theory the control can be understood as operating timetable, priorities of transitions, probabilities of transitions or the queues regulations, etc.). As to the structure, it consists of the number of operating nodes, the number of seats in the queue, etc.

Such division is not the only one and in special cases may be different.

The work presents the examples of processes model in multiphase systems of logistics operation. Incoming notifications of these systems are the processed objects. The operating nodes can be units and the systems of loading, packing, selection, etc.

2. THE EQUATIONS DESCRIBING THE SYSTEM

2.1 The discrete case (Markov chain)

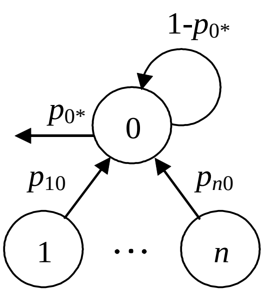

Fig. 1. A state in a discreet system

Let's analyze the graph shown in figure 1. Probability p_{0*} is the total probability of output from the state 0.

For the graph following equations occur.

The probability of state 0 in the next step, $\Delta t = 1$ (total probability):

$$p_0(k+1) = (1 - p_{0*})p_0(k) + \sum_{i=1}^{n} p_{i0}\,p_i(k) \tag{1}$$

$$\Updownarrow$$

$$p_0(k+1) = p_0(k) - p_{0*}p_0(k) + \sum_{i=1}^{n} p_{i0}\,p_i(k)$$

$$\Updownarrow$$

The change of probability in one step:

$$p_0(k+1) - p_0(k) = -p_{0*}p_0(k) + \sum_{i=1}^{n} p_{i0}\,p_i(k) \tag{2}$$

For the state provided the following equations occur.

$$p_0(k+1) - p_0(k) = 0, \qquad p_i(k) = p_i, \qquad i = 0,1,\dots,n \tag{3}$$

The proposition of a total probability (received from the equation (1) after consulting the equation (3)):

$$p_0 = (1 - p_{0*})p_0 + \sum_{i=1}^{n} p_{i0}\,p_i \tag{4}$$

$$\Updownarrow$$

The probability of output equals the probability of input (received from (2) after consulting the equation (3)):

$$p_{0*}p_0 = \sum_{i=1}^{n} p_{i0}\,p_i \tag{5}$$

2.2 The continuous case (Markov process)

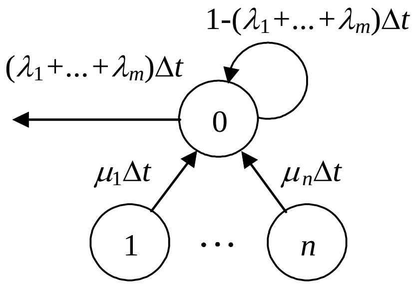

Fig. 2. A state in a continuous system

Let's analyze the graph shown in figure 2. In the state 0 the operation takes place in m nodes which are subjected to exponential distributions with coefficients $\lambda_1,..., \lambda_m$
For the graph presented, the following equations occur.

The probability of state 0 at the next moment (total probability):

$$p_0(t+\Delta t) = (1-\sum_{i=1}^{m}\lambda_i \Delta t) p_0(t) + \sum_{i=1}^{n}\mu_i \Delta t p_i(t) \quad (6)$$

$$\Updownarrow$$

$$\frac{p_0(t+\Delta t) - p_0(t)}{\Delta t} = -\sum_{i=1}^{m}\lambda_i p_0(t) + \sum_{i=1}^{n}\mu_i p_i(t)$$

$$\Updownarrow$$

The derivative of probability ($\Delta t \to 0$):

$$p_0'(t) = -\sum_{i=1}^{m}\lambda_i p_0(t) + \sum_{i=1}^{n}\mu_i p_i(t) \quad (7)$$

For the state provided the following equations occur.

$$p_0'(t) = 0, \qquad p_i(t) = p_i, \qquad i = 0,1,..., n \quad (8)$$

The proposition of a total probability (received from the equation (6) after consulting Δt =1 and the equation (8)):

$$p_0 = (1-\sum_{i=1}^{m}\lambda_i) p_0 + \sum_{i=1}^{n}\mu_i p_i \quad (9)$$

$$\Updownarrow$$

The probability of output equals the probability of input (received from the equation (7) after consulting the equation (8)):

$$\sum_{i=1}^{m}\lambda_i p_0 = \sum_{i=1}^{n}\mu_i p_i \quad (10)$$

The equations (1) (6), (2) (7), (4) (9), (5) (10) are their countertypes.

The equations (7) are called Chapman -Kolmogorov equations.

3. GEOMETRIC DISTRIBUTION

In discrete models the operating time will be modeled by discrete distributions.
Let's assume that $F(t) = p(\tau \le t) = 1 - p(\tau > t) = 1 - B(t)$
The discrete distribution has the following form:

$$B(t) = \begin{cases} 1 & 0 \le t < t_1 \\ 1 - \sum_{i=1}^{j} p_i & t_j \le t < t_{j+1}, \quad j = 1,2,...,k-1 \\ 0 & t_k \le t \end{cases} \tag{11}$$

Density function:

$$f(t) = \sum_{i=1}^{k} p_i \delta(t_i - t) \tag{12}$$

where: $t_i < t_j$ for $i < j, \quad i, j = 1,2,..., k$.
The probability of the appearance of notification in time t_i is p_i.
There is also: $\sum_{i=1}^{k} p_i = 1$
If $p_1 = 1$ we get determined distribution.

A special case of a discrete distribution is a geometric distribution for which:

$$f_\pi(t) = (1-\pi)\sum_{i=1}^{\infty} \pi^{i-1}\delta(iT - t) \tag{13}$$

$$B_\pi(t) = \pi^i \qquad iT \le t < (i+1)T, \quad i = 0,1,... \tag{14}$$

where: T geometric distribution period,
π geometric distribution parameter.

For the geometric distribution when we consider the specific moment in which the occurrence is to appear (time multiple T) it will appear with the probability 1-π and will not appear with the probability π. Such probability does not depend on whether a previous notification had appeared or not.
The mean value and the variance of geometric distribution are:

$$m_\pi = \frac{T}{(1-\pi)} \tag{15}$$

$$\sigma_\pi^2 = \frac{T^2}{(1-\pi)^2}\pi = m_\pi^2 \pi \tag{16}$$

If we approximate the distribuer of geometric distribution $B_\pi(t)$ exponential distribution (for the exponential distribution $B(t) = e^{-\lambda t}$) we get

$$e^{-kT\lambda} = \pi^k \quad \Rightarrow \quad \lambda = -\frac{\ln\pi}{T} \tag{17}$$

In borderline case where $\pi \to 1$ and $T \to 0$ so that λ=const geometric distribution aims at exponential distribution with parameter λ.
Geometric distribution is a discrete equivalent of the exponential distribution. For the geometric distribution just like for the exponential one, the distribution of the time to the nearest appearance of occurrence is not dependent on the waiting time (it is without the memory), because

$$P\{\tau > (k+r)T / \tau > kT\} = \frac{P\{\tau > (k+r)T \wedge \tau > kT\}}{P\{\tau > kT\}} =$$

$$= \frac{P\{\tau > (k+r)T\}}{P\{\tau > kT\}} = \frac{\pi^{k+r}}{\pi^k} = \pi^r = P\{\tau > rT\} \tag{18}$$

If we have two geometrical streams with two identical periods and at least one common notification point, so:
- the probability that in the moment kT there will not happen any of them

$$P(\tau > kT) = P(\tau_1 > kT)P(\tau_2 > kT) = (\pi_1\pi_2)^k \tag{19}$$

where: k = 0,1,2,...
- the probability that they will happen at the same time is

$$f_{\pi 1}(kT) f_{\pi 2}(kT) = (1-\pi_1)(1-\pi_2)(\pi_1\pi_2)^{k-1} \tag{20}$$

where: k = 1,2,3,...
what may be different from 0 (for the exponential distributions = 0).

4. THE OPERATING PARAMETERS

Before presenting exemplary models of operating systems, the operating parameters which the researcher may find interesting will be discussed.

First of all, these are the parameters which characterize the efficiency of a system. They include the average time being operated by a single notification system S (which means the average time interval between the moments of output from the system of two other notifications). More information can be found in the function of distribution of time intervals between the moments of the output from the system of two other notifications. However, it is not always possible to determine it by analysis. The average length of time of the presence of notification in a system R is also a very important parameter. The time of notification’s presence in a system is counted from the moment of notification’s input to the system till the moment of finishing operating of such notification by the last phase.

The length of staying and operating one notification in a system may be identical only in trivial case when the operation is not happening at the same time (when there is no more than one notification in a system at any time). Such an operating variant occurs in simple device.

The structure of the logistics system with the stream of flowing goods may consist of several operating phases. In such a system the next notification does not have to wait until the whole system will be released and can be operated as soon as the first phase of operation was released (so called pipelining).

Let's analyze the system consisting of *k* operating phases shown in figure 3.

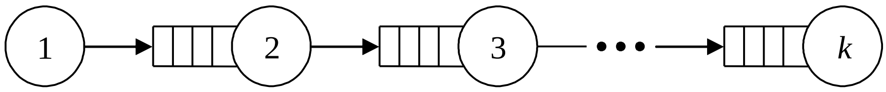

Fig. 3. The system consisting of *k* operating phases

Let's assume that the average operating time in phase *i* (*i* =1,...,*k*) is m_i.

For such a system with pipelined (*P*) and non pipelined (*N*) operating, the following relationships occur:

$$S_P \leq S_N = \sum_{i=1}^{k} m_i = R_N \leq R_{P\,i} \tag{21}$$

where: *S* – the average operating time of one notification
R – the average time of notification's presence in a system

and

$$S_P \geq \max_{i=1,2,\ldots k} m_i \tag{22}$$

$$R_P = t_{\text{wyjś}} - t_{\text{wejś}} = R_N + \Delta R \tag{23}$$

where: ΔR the total average time of notification spent in queues

The parameter S_P is strictly associated with max m_i which is the average time of the slowest operating phase (so called bottleneck of the system).

Both S_P and R_P are dependent on the structure of the system.

In the transition to pipelining operating *S* decreases because the time of unused appliances decreases as well because the notification is waiting in queues. These two operating parameters allow to expose the reserves of the system's efficiency. So:

$$\Delta S = S - \max m_i \tag{24}$$

shows the possible time of reserve that is lost by inappropriate operating rule at the given stream of notifications.

The time of notification spent in queues determines a reserve by which we can reduce the time of notification spent in a system. This reserve is:

$$\Delta R = R - \sum_{i=1}^{k} m_i \tag{25}$$

Another important parameter is the time r_i which the node i does not use in the operating process. The node cannot use the whole time on operating for two reasons. Firstly, it does not operate the notifications as there are no notifications on its inputs (the node is free). Secondly, it does not operate notifications as there is no place in the queue which accepts notifications being operated by this node (the node is blocked).

The components of time lost by the nodes having the maximum value m_i, are particularly important as their decrease gives the possibility of decreasing the average operating time of one notification. This time can be also decreased by decreasing the maximum value m_i. In each particular case determining the best way of decreasing S depends on the costs.

Other important parameter of system's operating is the probability g_ν that at any moment of time in a system there are ν notifications. The parameter g_ν characterizes so called operating depth.

The characteristic of the efficiency of the queue in a system may be the distribution of a random variable defining the level of saturation of the queue at any point in time.

The conditions of stationarity in the system as well as the impact of engaged rate on the characteristic of the processes within a system are also very important.

Let's discuss the possibilities of determining the operating parameters on the assumption that it is possible to obtain in any way the probability p_i that the system in a stationary state there is at any moment of time in the state i, where i takes the value of the numerable set of possible states.

To determine the average operating time S of one notification, let's choose an operating phase through which all the notifications go once only. In fact, in the queuing systems in which pipelining operation takes place, such phases are always present. Let m^* be the average operating time of notification in this phase. Let D be the set of states in which the system performs the operating notification in the selected phase. The value

$$p_D = \sum_{i \in D} p_i \tag{26}$$

shows the probability that the operation takes place in the selected phase.

For sufficiently large range of the time T with the probability close to 1 the average working time of the selected phase is:

$$T^* = p_D T \tag{27}$$

Then the average number of notifications that have been handled by the system at this time is equal to:

$$N^* = \frac{p_D T}{m^*} \tag{28}$$

and the average operating time of one notification by the system is defined by:

$$S = \frac{T}{N^*} = \frac{m^*}{p_D} \tag{29}$$

the distribution of operating depth is:

$$g_v = \sum_{i \in D_v} p_i \qquad v = 1,2,3,... \tag{30}$$

where: D_v set of states which are in the notification system v
Similarly, there are the distributions of the size of queue's saturation

The average number of notifications in a system

$$\bar{v} = \sum_{v=1}^{\infty} v g_v \tag{31}$$

The average time R of notifications' presence in a system can be defined with the usage of operating depth.

$$R = \bar{v} S \tag{32}$$

Now we will conduct the proof of equality (5.12).

THE PROOF:

There will be given an operating system with one input and one output for which there is a stationary state. Let's consider sufficiently large range of time T. Let M be the total average time of all notifications' presence in a system in time T. This is an analogy of energy, in our case dedicated to operation by the system. M can be determined in two ways.

$$M = \bar{v} T \tag{33}$$

$$M = RT / S \tag{34}$$

T / S is the average number of notifications being operated by the system in time T.

After comparing the last two relationships we get the equation (5.12).

It is worth noticing that any additional assumptions are not needed (e.g. the rules of the queues, the distribution of the input stream, the distributions of operating time, the priorities of entries inside the system). It is even acceptable that some notifications after their input into the system would never leave it (there can be finitely many such notifications due to the assumptions of the existence of a stationary state).

Parameter such as the function of distribution of length of time operating one notification in a system can be found by examining the transitional processes in SK.

5. THE MODEL AND ITS ANALYSIS

Let's consider two-phase operating system. The first node generates the notifications. The generation time is constant and equal 2. The second node operates the notifications. The operating time is random and is subjected to geometric distribution of the period T=1 and the parameter π. After the operation in the second node, the notification leaves the system. Between the phases there is a queue with a capacity of n notifications.

A node of the first phase may represent the generating device. A node of the second phase may represent a device selecting generated objects.

The states of our system will be denoted by threesome $<jt_1t_2>$.

The vector time component of such a system contains two parameters: t_1 the time remaining until the end of operating in the node of the first phase, t_2 the time remaining until the possible end in the node of the second phase. The component t_1 may have three values: 0 when the first node is blocked, 1 and 2 when it is working. The component t_2 may have two values: 0 when the second node is free, and 1 when it is working.

Combinational component of the vector's state has one component j which is the number of notifications in the queue, j=0,1,...,n.

The elements of matrix transitional probabilities will be determined by considering the possible transitions from the state n01 in which the first node is blocked, i.e. t_1=0. From this state with the probability 1-π the system goes to the state n21, when the second node finished notification's operation and took from the queue the next notification to operate, and the first blocked node passed the notification to the vacant place in the queue and started generating the next notification. With the probability π the second node will continue to operate the notification and the system will remain in the same state. From the state n21 the system may go to two states n-111 with the probability 1-π and to n11 with the probability π. Analogical transitions are possible from states i21, i=1,2,...,n so we have

$$\left.\begin{aligned} p[i21 \rightarrow i-111] &= 1-\pi \\ p[i21 \rightarrow i11] &= \pi \end{aligned}\right\} \tag{35}$$

where: i=1,2,...,n

From the state n11 the system with the probability π returns to the state n01 (the second node did not finish notification's operation and the first was blocked because there was no place in the queue), with the probability 1-π the system goes to state n21 (the nodes finished operating at the same time and started to operate other notifications, there was no change in the queue's).

For the state i11, i=0,1,...,n-1 the probability of transitions equals

$$\left.\begin{aligned} p[i11 \rightarrow i21] &= 1-\pi \\ p[i11 \rightarrow i+121] &= \pi \end{aligned}\right\} \tag{36}$$

From the state 021 the system with the probability π goes to the state 011, and with the probability 1-π to the state 010 (the second node is free and is waiting for notification).

Let's notice that the state 020 is not achievable from any state.

The graph of passes of the analyzed system is shown in figure 4.

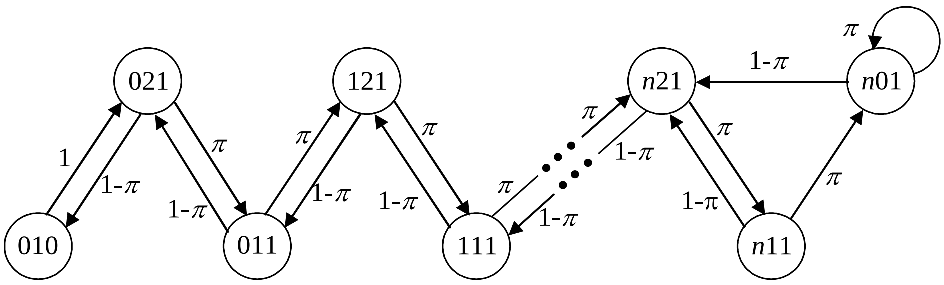

Fig. 4. The graph of a two-phase system. The first phase generates the notifications every two units of time. The second phase is subjected to the geometric distribution

The layout of equations defining the stationary probabilities of the analyzed system is:

$$\left.\begin{aligned}
&p_{010}=(1-\pi)\,p_{021}\\
&p_{021}=p_{010}+(1-\pi)\,p_{011}\\
&p_{i21}=\pi\,p_{i-111}+(1-\pi)\,p_{i11},\quad i=1,2,...,n-1\\
&p_{n21}=\pi\,p_{n-111}+(1-\pi)\,p_{n11}+(1-\pi)\,p_{n01}\\
&p_{i11}=(1-\pi)\,p_{i+121}+\pi\,p_{i21},\quad i=0,1,...,n-1\\
&p_{n11}=\pi\,p_{n21}\\
&p_{n01}=\pi\,p_{n01}+\pi\,p_{n11}
\end{aligned}\right\}\tag{37}$$

Assuming that $\pi \neq 1$ we may predict:

$$p=p_{010},\quad \omega=\pi/(1-\pi)\tag{38}$$

The solution of the layout is as following:

$$\begin{aligned}
&p_{i21}=\frac{\omega^{2i}}{1-\pi}\,p && i=0,1,...,n\\
&p_{i11}=\frac{\omega^{2i+1}}{1-\pi}\,p && i=0,1,...,n-1\\
&p_{n11}=\omega^{2n+1}\,p && p_{n01}=\omega^{2n+2}\,p
\end{aligned}\tag{39}$$

After taking into account the condition of normalization $\sum p_{ijk}=1$ from (6.26) we get

$$p=[1+\frac{1}{1-\pi}\sum_{i=0}^{2n}\omega^{i}+\omega^{2n+1}+\omega^{2n+2}]^{-1}\tag{40}$$

which results in

$$\left.\begin{aligned}
&p=\frac{1-2\pi}{2(1-\pi)-\omega^{2(n+1)}} && \omega\neq 1\\
&p=\frac{1}{4n+5} && \omega=1
\end{aligned}\right\}\tag{41}$$

The probability that the system is working is 1-p. Because m_2=T/(1-π)= 1/(1-π) the average time spent by the system to operate one notification is:

$$\left.\begin{aligned} S &= \frac{m_2}{1-p} = \frac{2(1-\pi)-\omega^{2n+2}}{(1-\pi)(1-\omega^{2n+2})} \qquad \omega \neq 1 \\ S &= \frac{4n+5}{2n+2} \qquad\qquad\qquad\qquad\quad \omega = 1 \end{aligned}\right\} \tag{42}$$

6. THE SYSTEM'S OPTIMIZATION

The example of the system's optimization discussed in chapter 5 has been presented below. We consider the special case when ω=1 (π=1/2).

Let's assume that the producer of the system wants to make a decision on how many seats should the queue between the operating phases contain. For this purpose, we assume that we have the following knowledge:

- the dependence indicating the manufacturing costs of the system in the function of the number of seats in the queue

$$K(n) = \frac{1}{300}n + \frac{1}{3} \tag{43}$$

- the dependence indicating the property of the market which is the price of the system in the function of system's efficiency and in the function of number of seats in a queue

$$C(n) = \frac{1}{S} = \frac{2n+2}{4n+5} \tag{44}$$

As the function of aim we will assume the value of the profit which is

$$Z(n) = C(n) - K(n) \tag{45}$$

So we have

$$Z(n) = \frac{2n+2}{4n+5} - \frac{1}{300}n - \frac{1}{3} \tag{46}$$

We will treat n as a continuous variable (n>0). The function $Z(n)$ is of class C^1.

$$Z'(n) = \frac{2}{(4n+5)^2} - \frac{1}{300} = 0 \tag{47}$$

The solution of the above equation shows that the function $Z(n)$ has a positive maximum for n=4,87. Since we are only interested in the total n so n=4 or n=5.

Z(4)=0,1295

Z(5)=0,1300

which means that optimal n=5.

7. CONCLUSION

The theory of queues can be successfully applied to the modeling of logistics systems with pipelining. The tools used in the theory allow one to model and optimize complex systems in an effective way. The work presents the examples of such an analysis.

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PRZYKŁAD OPTYMALIZACJI WIELKOŚCI KOLEJKI W NIEDETERMINISTYCZNYM SYSTEMACH LOGISTYCZNYCH

W pracy został przedstawiony przykład niedeterministycznego modelu systemu obsługi oraz optymalizacji wielkości kolejki w takim systemie. Do modelowania wykorzystano modele Markowa [3][4][13][16][17]. W pracy przedstawiony został także sposób wyznaczania istotnych parametrów pracy systemu niedeterministycznego.

W pierwszym oraz drugim rozdziale pracy przedstawione zostały ogólne informacje na temat modelowania oraz przedstawione zostały równania, z których skonstruowane są niedeterministyczne modele systemów z przeliczalną liczbą stanów. Rozróżnić tu należy modele dyskretne oraz ciągłe w czasie. W rozdziale trzecim przedstawione zostały informacje na temat rozkłady geometrycznego. Rozkład ten jest elementem modelu przedstawionego w dalszej części pracy. W rozdziale czwartym przedstawione zostały definicje istotnych parametrów obsługi oraz wyprowadzone zostały te parametry. Szczególnie istotne są dwa parametry: średni czas obsługi jednego zgłoszenia oraz średni czas przebywania zgłoszenia w systemie. Udowodnione zostało także twierdzenie, na temat czasu przebywania zgłoszeń w systemie. W rozdziale piątym przedstawiony został model przykładowego niedeterministycznego systemu obsługi. W systemie tym obsługa odbywa się w sposób potokowy. W systemie pomiędzy dwoma węzłami obsługi znajduje się kolejka, od której rozmiaru zależy jego sprawność. W rozdziale szóstym dla przedstawionego wcześniej modelu wyznaczony został optymalny rozmiar kolejki. W tym celu przyjęte zostały zależności określające koszty wytworzenia systemu w zależności od wielkości kolejki oraz zależność określająca cenę systemu w funkcji sprawności.

Słowa kluczowe: optymalizacja, wielkość kolejki, niedeterministyczne systemy logistyczne, modele Markowa

DOI: 10.7862/rz.2013.mmr.36

Tekst złożono w redakcji: wrzesień 2013
Przyjęto do druku: wrzesień 2013