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THE IDENTIFICATION OF THE LONG-TERM DEPENDENCE IN THE SALE OF COMMODITIES

The paper presents the algorithm and interpretation of Hurst exponent. It has been used during the analysis of sales in selected enterprise. The Hurst exponent was calculated empirically and theoretically, then the data was compared to verify if the analyzed time series have long memory.

1. INTRODUCTION

The case study of the prognosis of the sale is a basic part of the system of the decision support of manager in every enterprise. It is an exit-size with relation to remaining prognoses in the enterprise. In spite of that in the prognostic case one does not obtain exact and faultless answers, this always however it delivers more the information than intuitive solution. To the construction of the prognosis one uses methods permitting to gain the additional information on the investigated occurrence and to detect regularities ruling with analysed case. Used prognostic models have so on target not so much a definition of the value of the case, but should help understand the reality, and by this extend the knowledge of decision-makers about future events. Works in the area of prognosing have however a sense then, when the investigated row has not the fate character. In the first stage of prognostic works is necessary therefore to state, whether between the succeeding datas and input data and departure value steps out dependence, whether they also are completely accidental. To the execution of such analysis it is possible to utilize R/S method[2].

Reviewing literature with difficulty to find examples of the use of the R/S analysis to the research of the sale of commodities in the enterprise. Drawn therefore test the execution of such analysis in the example-enterprise.

2. THE ESSENCE OF R/S METHOD

The bases of R/S analysis bases on the work of Einstein from the year of 1905 of explanatory the basis of the movements of Brown[3]. The movements of Brown have been well-known since the year of 1827 as chaotic movements of very little particles hung in liquid. Each such movement is a exceptional event qualified as fluctuation. In the course

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[2] Tadeusiewicz R., Lula P., *Neuronal methods of the analysis of time series and possibilities of their uses in biomedical problems* , [in:] *The biocybernetics and the biomedical engineering 2000* ", under the ed. M. Nałęcz, the volume *Neural networks*, ed. of volume Wł. Duch, J. Korbicz, L. Rutkowski, R. Tadeusiewicz, PSA the Academical Publishing-Annexe WXIT, Warsaw 2000, s. 536

[3] Skoczylas K., *Hurst exponent application during the analysis of sale in an enterprise,* Econometrica 17, Wrocław 2006, s. 242-255

of time theory this became developed by Hurst giving the bases of R/S analysis. It permits on the calculation of the expression of Hurst, the element of the extended model of the movement of the particle moving with the movement of Brown. R/S analysis gives the possibility of the detection of long term dependences in time series, for which the standard methods of the statistical analysis did not prove any essential correlations[4]. The model given by Hurst for any ranks presents itself as follows:

$$\left(\frac{R}{S}\right)_n = cn^H \tag{1}$$

where:
R - the range of the accumulative time series,
S - the standard deviation, n - the number of the observation,
c - the positive constant,
H - the expression of Hurst

For the execution of the R/S analysis and the qualification of the expression of Hurst serves the following algorithm[5]:
1. The sequence of returns about the N length is necessary to share on the d of subsequences about the n length, so in order to $d \bullet n=N$,
2. For each subsequence is $m = 1, ..., d$ necessary:
a) to mark the average values of returns (E_m) and the empirical standard deviations (S_m);
b) to rescale values $Z_{i,m}$ by the deduction of the average value of returns in this subsequence

$$X_{i,m}=Z_{i,m} - E_m, \qquad \text{for } i=1,...,n; \tag{2}$$

c) to create the accumulative sequence of the rescaled returns

$$Y_{i,m} = \sum_{j=1}^{i} X_{j,m} \quad \text{for} \quad i=1,...,n; \tag{3}$$

d) to count the range: $R_m = \max \{Y_{1,m} ,..., Y_{n,m}\} - \min \{Y_{1,m} ,..., Y_{n,m}\}$;
e) to count value R_m/S_m for each analyzed subsequence
3. To count the average value rescaled range for the subsequences about the length n :

$$(R/S)_n = \frac{1}{d}\sum_{m=1}^{d}\frac{R_m}{S_m} \tag{4}$$

The Hurst expression is definite by the inclination of the lineal regression curve of points the log(R/S) in relation to the log(n) and can accept values from the section from 0 to 1. For the case of original movements of Brown H=0,5. The H size different from 0,5 marks that observations are not independent. Each observation keeps the memory about earlier events. This that happens presently is sent information in the future with the more and more smaller intensity, until the entire disappearance. Them larger H by this smaller

[4] Vandewalle N., Ausloos M., *Coherent and random sequences in financial fluctuations*, Physica A 246/1997, s. 454-459.
[5] Weron R., Przybyłowicz B., *Hurst analysis of electricity price dynamics*, Physica A 283/2000, s. 462-468.

dimension and level of risk that is to say steps out dependence conversely proportional. Hurst examined many natural occurrences. Almost all examined occurrences had H indeed greater from 0,5. For this type of time series Hurst accepted the time-limit „laden random walk", that marks joint trend with the noise. Researching along these occurrences the famous mathematician B. Mandelbrot called it „the fractional movements of Brown". Continuing these research scientists call today such time series fractal.

A serie of datas depending on the H value can be as follows interpreted:

- if H=0,5 marks this that the examined row reflects the stochastic (movements of Brown) case. The system remains at random, has the character of an accidental erring , the event are uncorrelated. This case is qualified also a name of the white noise;
- for 0<H<0,5 the system conquers the short cut than in the case of the fate erring, carrying out more frequent returns of the direction of translocation. This means that if in the given period the changes soar this is probable that in the next section will go down and on the contrary. Tendency this are all the more distinct them value H are nearer 0;
- for the case 0,5<H<1 row has characteristics strengthening a trend. In this area one observes fractal characteristcs of the row. Am characterized it by the effect of prolonged memory that is to say large degree of positive correlation. Theoretically, this what will become today, for ever influences on the future. If in the past stepped out growth trend, this exists chance, that this trend will be held also in the future. To them the higher H value to these smaller roaring of the row and the more strongly visible trend. In this section the examined row is qualified also a name of persistent.

Hurst leant own model on performance originating from research of the level of Nile of the relative of a several hundred year period, could therefore carry out analysis with the large exactitude, for the n large. Often steps out however lack of a high figure of datas and analysis conducts on the smaller test. In the case of the small n the literature[6] gives that the occurrence of the long-term memory takes place then, when appointed experientially the H value is greater at least $\sqrt{1/N}$ from the theoretical H value. In year of 1976 Anis and Lloyd they proposed the example, which modified Hurst example for the small n:

$$E(R/S)_n = \begin{cases} \dfrac{\Gamma(\frac{n-1}{2})}{\sqrt{\pi}\,\Gamma(\frac{n}{2})} \sum_{i=1}^{n-1} \sqrt{\dfrac{n-i}{i}} & \text{dla } n \le 340, \\ \sqrt{\dfrac{2n}{\pi(n-1)^2}} \sum_{i=1}^{n-1} \sqrt{\dfrac{n-i}{i}} & \text{dla } n > 340. \end{cases} \tag{5}$$

In the year 1994 Peters introduced the correction to the introduced above formula, because on the base by himself the computer simulation he ascertained that for n<20 the

[6] Jajuga K., Papla D., *The theory of the chaos in the analysis of financial time series - theoretical aspects and empirical research*, V the All-Polish Scientific Seminar, the Nicholas Kopernik University in Toruń, Toruń 1997; Weron A., Weron R., *The financial engineering: the pricing of tools of derivatives, the computer simulation, the statistics of the market*, Warsaw 1998, s. 326

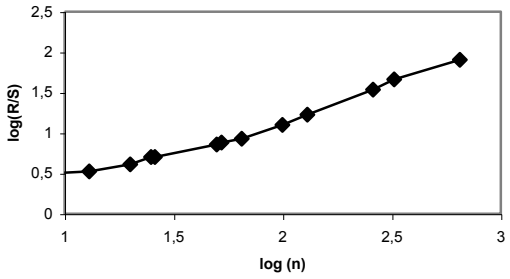
Anis and formula caused the inflatedness of the value[7]. Proposed by him the formula assured for any n the possibility of the H enumeration of theoretical:

$$E(R/S)_n = \begin{cases} \dfrac{n-0{,}5}{n} \dfrac{\Gamma(\frac{n-1}{2})}{\sqrt{\pi}\Gamma(\frac{n}{2})} \sum_{i=1}^{n-1} \sqrt{\dfrac{n-i}{i}} & \text{dla } n \le 340, \\ \dfrac{n-0{,}5}{n} \sqrt{\dfrac{2n}{\pi(n-1)^2}} \sum_{i=1}^{n-1} \sqrt{\dfrac{n-i}{i}} & \text{dla } n > 340. \end{cases} \quad (6)$$

3. THE USE OF THE R/S METHOD IN THE ANALYSIS OF THE SALE OF COMMODITIES

The R/S method is one of more time-consuming long-term analyses, thereby for its execution one prepared the computer programme. From the assumptions of method results, that to them substantial amount the observation is surrendered to the research, the all the more exact one receives results. To the analysis of the sale of commodities in the examined enterprise one accepted historical datas for the period of 3,5 year. Research with the utilization of the R/S method one effected with reference to the sale generality and assortment groups. Research were begun from the analysis of the sale of commodities generality. It is a base to the elaboration of schedules from now on and treats of the different type of the decision. The results of the R/S analysis R/S one introduced in fig. 1.

Fig. 1. The R/S analysis for the value of the day′s sale in the enterprise



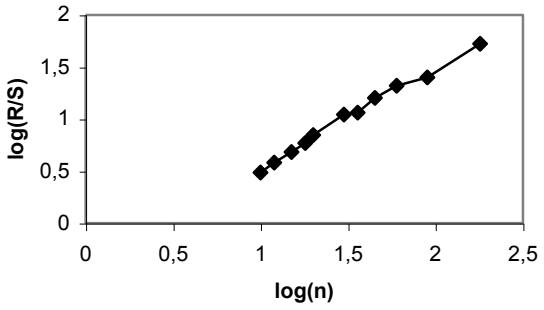
Source: My elaboration

The value of the expression of Hurst carried out H=0,797922, what considerably exceeds 0,5. Calculated for needs of the present analysis the theoretical value of the expression of Hurst carried out H=0,568, instead $\sqrt{1/N}$ =0,027. Comparing the empirical value with theoretical it is visible that for real sale values appears the effect of the prolonged

[7] Purczyński J., *The chaos and the R/S analysis*, Conference-Materials of the Szczecin University, 2000, s. 131

memory. Events of the past have the influence on this, what happens at present in the case of the sale of commodities. Such occurrence suggests the occurrence of trends. Exists so the probability (approx. 80% the chance) that the at present existing trend will remain also in the future. The H value does not show however on the kind of the trend.
On the ground effected analysis one can ascertain that in the case of the examined sale of commodities generality the past bears on the present, and the present will have an influence from now on. There appears so the connection among the order and sometimes occurrent events.
To the R/S analysis one surrendered also ranks representing the weekly value of the real sale of commodities. Results of the analysis introduced in fig. 2.

Fig. 2. The R/S analysis for the weekly sale of commodities generality in the mercantile agency



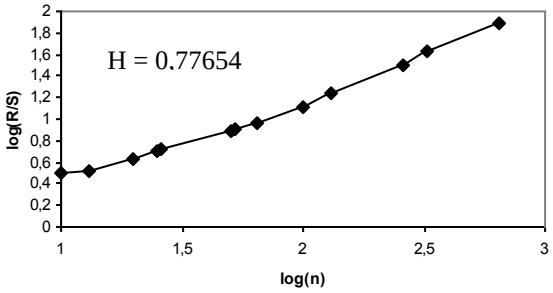
Source: My elaboration

The expression carried out H=0,972762. Is is the higher value from H of enumerated for the daily sale. One counted also H theoretical whose the equal value is 0,6, and $\sqrt{1/N}$ =0,0745. Passed calculations doubtless testify about the occurrence of the high dependence between the succeeding datas.
On the base passed analysis for the weekly sale one can ascertain that the greater H value for the more and more smaller frequency of the measurement can testify about smaller roaring of the row. Very high H speaks that at such temporary prospects the influence of fate disturbances stops. One can so infer that to the analysis of the long-term dependence it is better to accept ranks consequential from measurement of less frequent, but more long-drawn-out during.

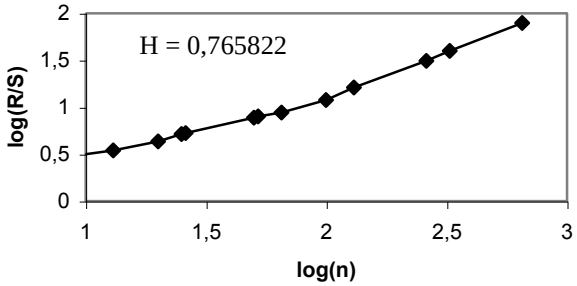
A following stage of research was the execution of the R/S analysis of groups for 15 assortment groups occurrent in analysed mercantile agency. Results of this R/S analysis one introduced in fig. 3.

Fig. 3. The R/S analysis for the sale of day′s in the assortment group: a) alcohols, b) chemistry, cosmetics, c) other, d) coffee, tea, e) tinned food, preserves, f) spices, the flours and cerealses, g) the meat, pork products, fishes, h) frozen food, frozen fishes, i) beverages, juices, j) fruits and vegetables, k) cigarettes, l) the dairy produce, the bread, ł) the beer, m) wine, n) confectionery

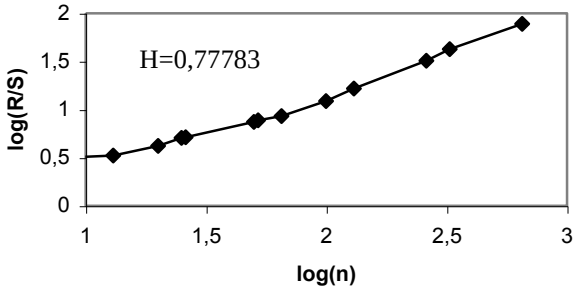
a)



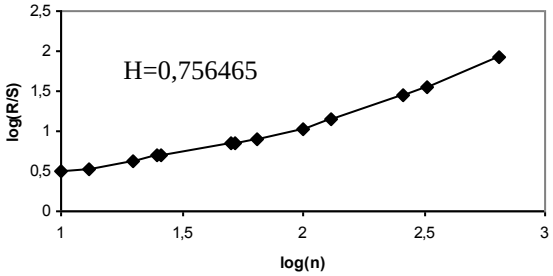
b)



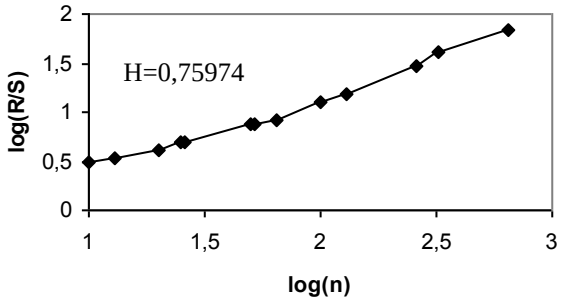
c)



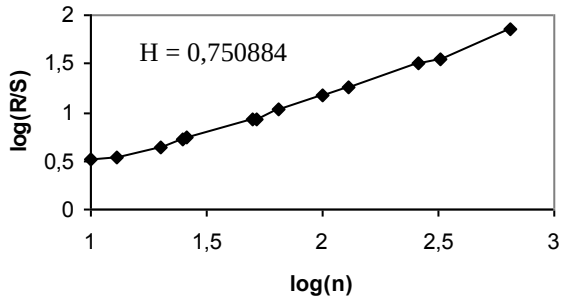
d)


H=0,756465
log(R/S)
log(n)
2,5
2
1,5
1
0,5
0
1
1,5
2
2,5
3

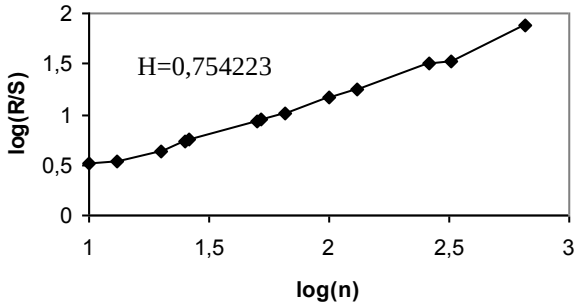
e)


H=0,75974
log(R/S)
log(n)
2
1,5
1
0,5
0
1
1,5
2
2,5
3

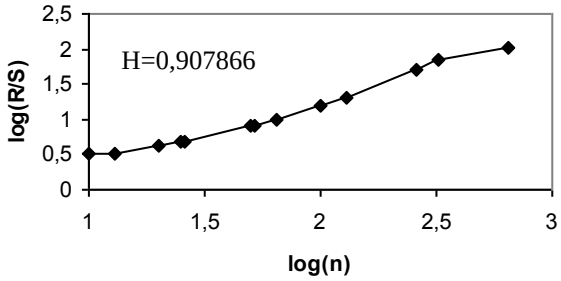
f)


H = 0,750884
log(R/S)
log(n)
2
1,5
1
0,5
0
1
1,5
2
2,5
3

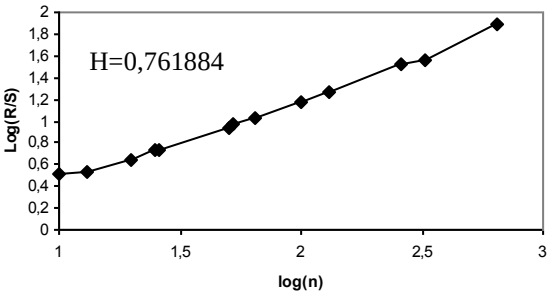
g)


H=0,754223
log(R/S)
log(n)
2
1,5
1
0,5
0
1
1,5
2
2,5
3

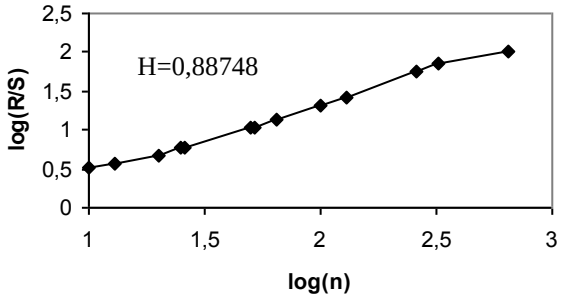
h)


H=0,907866
log(R/S)
log(n)
2,5
2
1,5
1
0,5
0
1
1,5
2
2,5
3

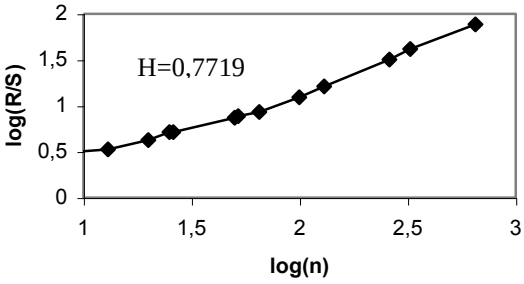
i)


H=0,761884
Log(R/S)
log(n)
2
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
1,5
2
2,5
3

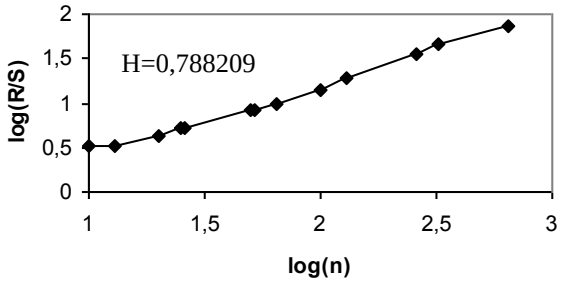
j)


H=0,88748
log(R/S)
log(n)
2,5
2
1,5
1
0,5
0
1
1,5
2
2,5
3

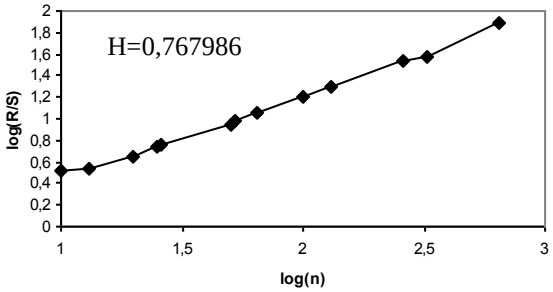
k)


H=0,7719
log(R/S)
log(n)
2
1,5
1
0,5
0
1
1,5
2
2,5
3

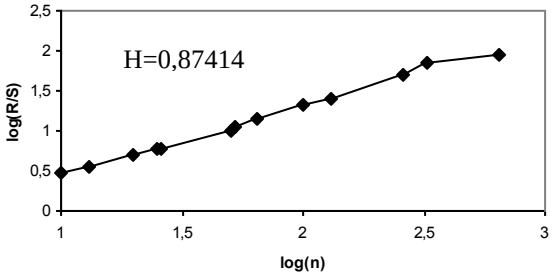
l)


H=0,788209
log(R/S)
log(n)
2
1,5
1
0,5
0
1
1,5
2
2,5
3

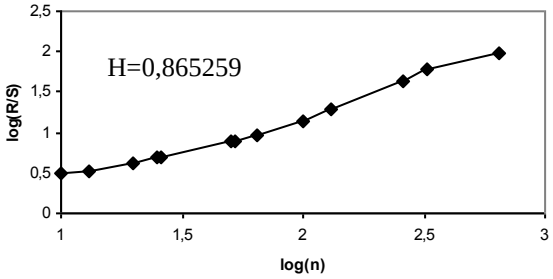
ł)



m)



n)



Source: My elaboration

To the analysis of the real day′s sale for assortment groups one used 1300 observations which one divided on such themselves subsequences as at the sale generality. For all analysed groups of goods the value of the expression of Hurst considerably crosses 0,5, be-

cause fluctuated within from H=0,750884 to H=0,907866. Comparing empirical values of the expression with the theoretical (*H=0,568,* $\sqrt{1/N}$ *=0,0745*) value one can the resoluteness ascertain that appears the data dependence in stock all assortment groups.

4. THE END

Summing up one can say that the expression of Hurst is a excellent measure for time series. Takes into account the order of the observation arow. This characteristic is as most desirable in the analysis of financial time series, where one of main assignments is the detection of the dependence between following observations. Passed herein to the elaboration the analysis showed the occurrence of the long-term dependence, both for the sale of commodities generality, as and the sale in all assortment groups. On the base of these information it is possible to conclude about the possibility of surrender in the more distant researched row to prognosing.

LITERATURE

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[4] Tadeusiewicz R, Lula P., *Neuronal methods of the analysis of time series and possibilities of their uses in biomedical problems* , [in:] *The biocybernetics and the biomedical engineering 2000* ", under the ed. M. Nałęcz, the volume *Neural networks*, ed. of volume Wł. Duch, J. Korbicz, L. Rutkowski, R. Tadeusiewicz, PSA the Academical Publishing-Annexe WXIT, Warsaw 2000.

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IDENTYFIKACJA ZALEŻNOŚCI DŁUGOTERMINOWEJ W SPRZEDAŻY TOWARÓW

Artykuł przedstawia sposób postępowania i interpretacji wykładnika Hursta. Został on zastosowany do analizy sprzedaży w wybranym przedsiębiorstwie. Obliczono teoretyczną i empiryczną wartość wykładnika Hursta, a następnie porównano go i zweryfikowano na podstawie analizy szeregów czasowych.

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