ZESZYTY NAUKOWE POLITECHNIKI RZESZOWSKIEJ 290, Mechanika 86
RUTMech, t. XXXI, z. 86 (4/14), październik-grudzień 2014, s. 589-602

Gennadij SHUSHKEVICH[1]
Svetlana SHUSHKEVICH[2]

THE PENETRATION OF THE SOUND FIELD OF THE SPHERICAL RADIATOR THROUGH THE PLANE ELASTIC LAYER

In this paper the results of exact solution of the axisymmetric problem of the penetration of the sound field through the plane elastic layer are presented. The spherical radiator is located in a thin unclosed spherical shell as the source of the acoustic field. Using appropriate theorems, the solution of the boundary conditions problem is reduced to solve dual functions in Legendre's polynomials, which are converted to the infinite system of linear algebraic equations of the second kind with a completely continuous operator. The influence of some parameters of the problem on the value of the coefficient of shielding sound field is investigated.

Keywords: elastic plate, sound field, spherical radiator

1. Introduction

The research of the distribution of the sound waves in elastic environment has a great importance in medical diagnostics, in the underwater acoustics and in the seismology, etc. [1-3]. Sandler and Maev [4] considered the problem of calculating the propagation of acoustic waves within an ideal isotropic multilayer plate structure. Exploring this problem by examining the ray paths of the multiple reflections within the plate structure, it is possible to show that upon careful consideration many of these paths will travel equivalent distances in time and space becoming coincident. The solution of a problem of dispersion of a spherical sound wave on a multilayered uniform firm plate can be consolidated to system of the algebraic equations [5]. Results of research of distribution of sound waves in poorly connected acoustic layers with rigid borders are presented by Gortinskaja and Popov [6]. For the solution of the Helmholtz equation with Neumann's boundary conditions the method of coordination of asymptotic decomposition of solutions of regional problems is used.

1 Autor do korespondencji/corresponding author: Gennadij Shushkevich, Yanka Kupala State University of Grodno, 22 Ozheshko St., 230023 Grodno, Belarus, e-mail: g_shu@tut.by
2 Svetlana Shushkevich, Yanka Kupala State University of Grodno, e-mail: spusha@list.ru

Yan and Zhao [7] considered the inverse problem of the scattering of a plane acoustic wave by a multilayered scatterer. The inverse scattering problem is analysed as the problem of determining the shape of a multilayered scatterer by measurements of the far field patterns of acoustic or electromagnetic scattered waves. Transfer matrix technique is used by Vashishth and Gupta [8] to study the layered materials. The effects of frequency, porosity, angle of incidence, layer thickness and number of layers on the energy ratios and surface impedance are studied for different configurations of the layered materials. In recent study Kiselyova and Shushkevich [9] considered the solution of a problem on penetration of a sound field through of system permeable planes. As a source of a field the spherical radiator located in a thin not closed spherical cover is considered. The layers of the plate are made up of ideal acoustic materials (linear, homogeneous, isotropic, non dispersive) with known material parameters, and the plates are assumed to be bonded such that the interfaces follow the perfectly bonded boundary conditions [10, 11].

The aim of the paper is construct the exact solution of the axisymmetric problem of the penetration of the sound field through the flat elastic layer. The influence of some parameters of the problem for the value of the coefficient of shielding sound field is investigated.

2. Problem formulation

Let all the space R^3 be spitted by planes $S_0\left(z=h_1\right)$ and $S_1\left(z=h_1+h_2\right)$ on the fields $D_0(z<h_1)$, $D_2(h_1<z<h_1+h_2)$, $D_1(z>h_1+h_2)$ (fig. 1.). The area D_0 has thin unclosed spherical shell Γ_1 perfectly, located on the sphere Γ of the radius a with the center at the point O. We denoted $D_0^{(0)}(0\le r<a)$ the area of space bounded by the sphere Γ and $D_0=D_0^{(0)}\cup\Gamma\cup D_0^{(1)}$. The distance between points O and O_1 is equal h_1, h_2 is the distance between planes S_0 and S_1.

The point radiator of the sound waves oscillating with angular frequency ω is located at the point O. Areas D_j, j = 0.1, filled with a material in which shear waves do not propagate *A* density of the medium and a speed of sound in the area D_j are denoted by $\tilde{\rho}_j$, c_j, respectively. The area D_2 is a plane elastic layer. The elastic layer oscillates under the influence of the sound field. Its deformation is determined by the displacement vector $\vec{u}$ that satisfies the Lame equation [2, 3]:

$$\tilde{\mu}\Delta\vec{u}+(\tilde{\lambda}+\tilde{\mu})\operatorname{grad}\operatorname{div}\vec{u}+\omega^2\tilde{\rho}\vec{u}=0 \tag{1}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator, $\tilde{\lambda}$, $\tilde{\mu}$ are Lame coefficients, $\tilde{\rho}$ is density of the medium.

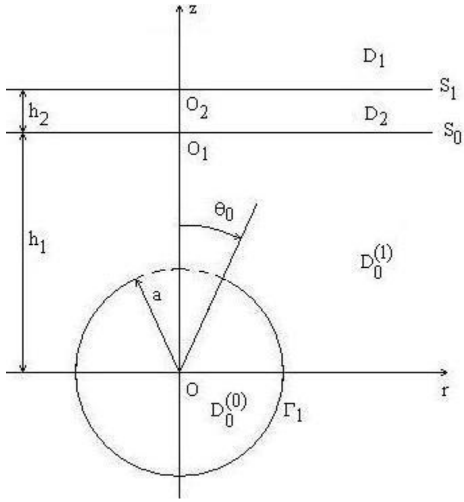

Fig. 1. Geometry of the problem

To solve this problem we connected spherical coordinates $\{r,\ \theta,\ \varphi\}$ and cylindrical coordinates $\{\rho,\ \varphi,\ z\}$ with the point O. The spherical shell Γ_1 and the plane S_j, j = 0, 1 are described as follows:

$$\Gamma_1 = \{r = a,\ \theta_0 \le \theta \le \pi,\ 0 \le \varphi \le 2\pi\} \tag{2}$$

$$S_0 = \{z = h_1,\ 0 \le \rho \le \infty,\ 0 \le \varphi \le 2\pi\} \tag{3}$$

$$S_1 = \{z = h_1 + h_2,\ 0 \le \rho \le \infty,\ 0 \le \varphi \le 2\pi\} \tag{4}$$

Let p_c be the pressure of the sound field of the primary point radiator, $p_0^{(0)}$ be the secondary sound pressure field in the area $D_0^{(0)}$, $p_0 = p_0^{(1)} + p_0^{(2)}$ be the secondary sound pressure field in the area $D_0^{(1)}$ and p_1 be the secondary sound pressure field in the area D_1. The actual displacement and the sound pressure are calculated by the formulas $\vec{U} = \mathrm{Re}\left(\vec{u}\,\mathrm{e}^{-i\omega t}\right)$ and $P_j = \mathrm{Re}\left(p_j\,\mathrm{e}^{-i\omega t}\right)$. P_j is imaginary unit. The pressures of secondary sound field $p_0^{(j)}$ (j = 0, 1, 2) and p_1 satisfies the Helmholtz equation [2, 3]:

$$\Delta p_0^{(j)} + k_0^2 p_0^{(j)} = 0 \quad \text{in} \quad \mathrm{D}_0, \quad \Delta p_1 + k_1^2 p_1 = 0 \quad \text{in} \quad \mathrm{D}_1 \tag{5}$$

where: $k_0 = \omega / c_0$, $k_1 = \omega / c_1$ are wave numbers.

The displacement vector is determined by the formula [2]:

$$\vec{\mathrm{u}} = \operatorname{grad} \psi + \operatorname{rot}\left(-\frac{\partial \Phi}{\partial \rho} \vec{\mathrm{e}}_\varphi \right) \tag{6}$$

The equation (6) is satisfied in the case of propagation of small disturbances in an elastic body for steady-state motion of the particles of the body. Functions ψ and Φ satisfy the Helmholtz equation and are defined as:

$$\left. \begin{aligned} &\Delta \psi + k_l^2 \psi = 0, \quad k_l = \omega / c_l, \quad c_l = \sqrt{(\tilde{\lambda} + 2\tilde{\mu}) / \tilde{\rho}} \\ &\Delta \Phi + k_t^2 \Phi = 0, \quad k_t = \omega / c_t, \quad c_t = \sqrt{\tilde{\mu} / \tilde{\rho}} \end{aligned} \right\} \tag{7}$$

where: $\tilde{n}_l$, c_t are velocity of longitudinal and transverse elastic waves respectively.

In cylindrical coordinate system components of the displacement vector are associated with the functions ψ and Φ by relations:

$$u_\rho = \frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \Phi}{\partial \rho \partial \mathrm{z}}, \quad u_z = \frac{\partial \psi}{\partial \mathrm{z}} + \frac{\partial^2 \Phi}{\partial \mathrm{z}^2} + k_t^2 \Phi \tag{8}$$

The solution of the diffraction problem is reduced to find the displacement vector $\vec{u}(u_\rho, u_z, 0)$, the pressures of the sound field $p_0^{(j)}$ (j = 0, 1, 2) p_1 which satisfy the boundary condition on the surface of the spherical shell (acoustically hard shell):

$$\frac{\partial}{\partial r} \left(p_c + p_0^{(0)} \right) \Big|_{\Gamma_1} = 0 \tag{9}$$

boundary conditions of the interaction of the sound field with an elastic layer on a plane S_j:

$$u_z \Big|_{\mathrm{S}_j} = \omega^{-2} \tilde{\rho}_j^{-1} \frac{\partial p_j}{\partial \mathrm{z}} \Bigg|_{\mathrm{S}_j}, \quad \frac{\partial u_\rho}{\partial \mathrm{z}} + \frac{\partial u_z}{\partial \rho} \Bigg|_{\mathrm{S}_j} = 0 \tag{10}$$

$$\left(2\tilde{\mu}+\tilde{\lambda}\right)\frac{\partial u_z}{\partial z}+\tilde{\lambda}\left(\frac{u_\rho}{\rho}+\frac{\partial u_\rho}{\partial \rho}\right)\Bigg|_{S_j}=-p_j\Big|_{S_j} \tag{11}$$

The condition at infinity [2, 3, 12] can be written as:

$$\lim_{M\to\infty} r\left(\frac{\partial p_j(\mathrm{M})}{\partial r}-ik_j p_j(\mathrm{M})\right)=0, \qquad j=0,\ 1 \tag{12}$$

where M is an arbitrary point at the space.

Condition at continuity of pressure on the open part of the spherical shell Γ/Γ_1 is given by:

$$\left(p_c+p_0^{(0)}\right)\Big|_{\Gamma\backslash\Gamma_1}=\left(p_0^{(1)}+p_0^{(2)}\right)\Big|_{\Gamma\backslash\Gamma_1} \tag{13}$$

and the normal derivative on the surface of the sphere Γ is:

$$\frac{\partial}{\partial r}\left(p_c+p_0^{(0)}\right)\Big|_{\Gamma}=\frac{\partial}{\partial r}\left(p_0^{(1)}+p_0^{(2)}\right)\Big|_{\Gamma} \tag{14}$$

The initial pressure of the sound field can be represented in the form [12]:

$$p_c(r,\theta)=P\exp(ik_0 r)/r=P\sum_{n=0}^{\infty} f_n h_n^{(1)}\left(k_0 r\right)P_n(\cos\theta), \quad f_n=ik_0\delta_{0n} \tag{15}$$

where $h_n^{(1)}\left(x\right)$ are spherical Hankel functions, $P_n(\cos\theta)$ is Legendre polynomi-nals [13], δ_{0n} is Kronecker delta and P is constant.

The pressure of the scattered sound field is represented as a superposition of basic solutions of the Helmholtz equation in spherical and cylindrical coordi-nates [14], taking into account the condition at infinity (12) we have:

$$p_0^{(0)}(r,\theta)=P\sum_{n=0}^{\infty} c_n j_n\left(k_0 r\right)P_n\left(\cos\theta\right) \quad \text{in} \quad \mathrm{D}_0^{(0)} \tag{16}$$

$$\left.\begin{aligned}&\sum_{n=0}^{\infty}\frac{x_n-f_n}{\dfrac{\mathrm{d}}{\mathrm{d}\xi_0}j_n\left(\xi_0\right)}P_n(\cos\theta)=0,\ \ 0\le\theta<\theta_0\\ &\sum_{n=0}^{\infty}x_n\frac{\mathrm{d}}{\mathrm{d}\xi_0}h_n^{(1)}\left(\xi_0\right)P_n\left(\cos\theta\right)=-\sum_{n=0}^{\infty}T_n\frac{\mathrm{d}}{\mathrm{d}\xi_0}j_n\left(\xi_0\right)P_n\left(\cos\theta\right),\ \ \theta_0<\theta\le\pi\end{aligned}\right\} \tag{17}$$

$$p_1(\rho,z) = P\int_0^\infty d(\lambda) J_0(\lambda\rho) \mathrm{e}^{-v_1(z-h_1-h_2)} \lambda d\lambda \quad \text{in} \quad \mathrm{D}_1 \tag{18}$$

$$\psi(\rho,z) = P\int_0^\infty \left(a(\lambda) \mathrm{e}^{-v_\ell(z-h_1)} + b(\lambda) \mathrm{e}^{v_\ell(z-h_1-h_2)} \right) J_0(\lambda\rho) \lambda d\lambda \tag{19}$$

$$\Phi(\rho,z) = P\int_0^\infty \left(\tilde{a}(\lambda) \mathrm{e}^{-v_t(z-h_1)} + \tilde{b}(\lambda) \mathrm{e}^{v_t(z-h_1-h_2)} \right) J_0(\lambda\rho) \lambda d\lambda \tag{20}$$

where $j_n(x)$ are spherical Bessel functions of the first kind, $J_0(x)$ are Bessel functions of the first kind, $v_j = \sqrt{\lambda^2 - k_j^2}$, $-\pi/2 \le \arg v_j < \pi/2$, $j = 0, 1$; $v_\ell = \sqrt{\lambda^2 - k_\ell^2}$, $-\pi/2 \le \arg v_\ell < \pi/2$, $v_t = \sqrt{\lambda^2 - k_t^2}$, $-\pi/2 \le \arg v_t < \pi/2$.

Unknown coefficients c_n, x_n and functions $a(\lambda)$, $b(\lambda)$, $\tilde{a}(\lambda)$, $\tilde{b}(\lambda)$, $y(\lambda)$, $d(\lambda)$ must be determined from boundary conditions.

3. Boundary conditions

The boundary conditions are defined by eqs. (1), (9) and (11). The function $p_0^{(2)}(\rho,z)$ through spherical wave functions, using the formula connecting cylindrical and spherical wave functions is:

$$\left. \begin{aligned} & J_0(\lambda\rho) \mathrm{e}^{vz} = \sum_{n=0}^\infty (-i)^n (2n+1) P_n\left(\frac{iv}{k}\right) j_n(kr) P_n(\cos\theta) \\ & v = \sqrt{\lambda^2 - k^2}, \ -\pi/2 \le \arg v < \pi/2 \end{aligned} \right\} \tag{21}$$

then

$$\left. \begin{aligned} & p_0^{(2)}(r,\theta) = P\sum_{n=o}^\infty T_n j_n(k_0 r) P_n(\cos\theta) \\ & T_n = (-i)^n (2n+1) \int_0^\infty y(\lambda) P_n\left(\frac{iv_0}{k_0}\right) \mathrm{e}^{-v_0 h} \lambda d\lambda \end{aligned} \right\} \tag{22}$$

According to eqs. (12)-(14) and eq. (17), the boundary condition (11) taking into account the condition of orthogonality of Legendre polynomials on the interval $[0; \pi]$ will become:

$$\left.\begin{array}{l} f_n \dfrac{\mathrm{d}}{\mathrm{d}\xi_0} h_n^{(1)}(\xi_0) + c_n \dfrac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0) = x_n \dfrac{\mathrm{d}}{\mathrm{d}\xi_0} h_n^{(1)}(\xi_0) + T_n \dfrac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0) \\ \xi_0 = k_0 a, \quad n = 0, 1, \ldots \end{array}\right\} \tag{23}$$

Let's specify the boundary condition (9) on the surface of a spherical shell and the condition of continuity (13). Let's exclude factors c_n in the resulting equations, using the eq. (23). Then dual equations in Legendre's polynomials take the form:

$$\left.\begin{array}{l} \displaystyle\sum_{n=0}^{\infty} \frac{x_n - f_n}{\dfrac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0)} P_n(\cos\theta) = 0, \quad 0 \le \theta < \theta_0 \\ \displaystyle\sum_{n=0}^{\infty} x_n \frac{\mathrm{d}}{\mathrm{d}\xi_0} h_n^{(1)}(\xi_0) P_n(\cos\theta) = -\sum_{n=0}^{\infty} T_n \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0) P_n(\cos\theta), \quad \theta_0 < \theta \le \pi \end{array}\right\} \tag{24}$$

Let a new coefficient be:

$$x_n = X_n \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0) + f_n, \qquad n = 0, 1, \ldots \tag{25}$$

and a small parameter be:

$$g_n = 1 + \frac{4i\xi_0^3}{2n+1} \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0) \frac{\mathrm{d}}{\mathrm{d}\xi_0} h_n^{(1)}(\xi_0) \tag{26}$$

Then we will make replacement $\theta = \pi - \tilde{\theta}$, $\theta_0 = \pi - \tilde{\theta}_0$, $\tilde{X}_n = (-1)^n X_n$ for the transformation of dual eqs. (19). As a result, dual eqs. (24) take the form:

$$\left.\begin{array}{l} \displaystyle\sum_{n=0}^{\infty} (2n+1)(1-g_n)\tilde{X}_n P_n(\cos\tilde{\theta}) = \sum_{n=0}^{\infty} (-1)^n (2n+1)(\tilde{f}_n + \tilde{T}_n) P_n(\cos\tilde{\theta}), \quad 0 \le \tilde{\theta} < \tilde{\theta}_0 \\ \displaystyle\sum_{n=0}^{\infty} \tilde{X}_n P_n(\cos\tilde{\theta}) = 0, \quad \tilde{\theta}_0 < \tilde{\theta} \le \pi \end{array}\right\} \tag{27}$$

where

$$\tilde{T}_n = 4i\xi_0^3 T_n \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_n(\xi_0) / (2n+1), \quad \tilde{f}_n = 4i\xi_0^3 f_n \frac{\mathrm{d}}{\mathrm{d}\xi_0} h_n^{(1)}(\xi_0) / (2n+1) \tag{28}$$

Dual eqs. (25) are converted to an infinite system of linear algebraic equations of the second kind with the completely continuous operator using the integral representation for the Legendre polynomials [15, 16]:

$$\tilde{X}_n - \sum_{k=0}^{\infty} g_k R_{nk} \tilde{X}_k = \sum_{k=0}^{\infty} (-1)^k \left(\tilde{T}_k + \tilde{f}_k\right) R_{nk}, \; n = 0, 1, ... \tag{29}$$

where

$$\left.\begin{array}{l} R_{nk} = \dfrac{1}{\pi}\left[\dfrac{\sin\left(n-k\right)(\pi-\theta_0)}{n-k} - \dfrac{\sin\left(n+k+1\right)(\pi-\theta_0)}{n+k+1}\right] \\ \left.\dfrac{\sin\left(n-k\right)(\pi-\theta_0)}{n-k}\right|_{n=k} = \pi - \theta_0 \end{array}\right\} \tag{30}$$

To satisfy boundary conditions (11), the function $p_0^{(1)}\left(r,\theta\right)$ through cylindrical wave functions takes the form:

$$\left.\begin{array}{l} h_n^{(1)}\left(kr\right) P_n\left(\cos\theta\right) = \displaystyle\int_0^{\infty} \frac{i^{-n-1}}{kv} P_n\left(\frac{iv}{k}\right) J_0\left(\lambda\rho\right) \mathrm{e}^{-vz} \lambda d\lambda \\ v = \sqrt{\lambda^2 - k^2}, \; -\pi/2 \le \arg v < \pi/2, \quad z > 0 \end{array}\right\} \tag{31}$$

then

$$p_0^{(1)}\left(\rho, z\right) = P\int_0^{\infty} Z\left(\lambda\right) J_0\left(\lambda\rho\right) \mathrm{e}^{-v_0 z} \lambda d\lambda, \quad Z\left(\lambda\right) = \frac{1}{k_0 v_0} \sum_{n=0}^{\infty} i^{-n-1} P_n\left(\frac{iv_0}{k_0}\right) x_n \tag{32}$$

Taking into account the eqs. (17)-(20) and (32) and boundary conditions (11), the linear algebraic equation takes the form:

$$M\left(\lambda\right)\cdot V\left(\lambda\right) = F\left(\lambda\right)\cdot Z\left(\lambda\right) \tag{33}$$

where

$$M(\lambda) = \begin{pmatrix} m_{11}(\lambda) & m_{12}(\lambda) & m_{13}(\lambda) & m_{14}(\lambda) & 1 & 0 \\ m_{21}(\lambda) & m_{22}(\lambda) & m_{23}(\lambda) & m_{24}(\lambda) & 0 & 0 \\ m_{31}(\lambda) & m_{32}(\lambda) & m_{33}(\lambda) & m_{34}(\lambda) & m_{35}(\lambda) & 0 \\ m_{41}(\lambda) & m_{42}(\lambda) & m_{43}(\lambda) & m_{44}(\lambda) & 0 & 1 \\ m_{51}(\lambda) & m_{52}(\lambda) & m_{53}(\lambda) & m_{54}(\lambda) & 0 & 0 \\ m_{61}(\lambda) & m_{62}(\lambda) & m_{63}(\lambda) & m_{64}(\lambda) & 0 & m_{66}(\lambda) \end{pmatrix},$$

$$V(\lambda)=\begin{pmatrix} a(\lambda)\\ b(\lambda)\\ \tilde{a}(\lambda)\\ \tilde{b}(\lambda)\\ y(\lambda)\\ d(\lambda)\end{pmatrix},\quad F(\lambda)=\begin{pmatrix} f_1(\lambda)\\ 0\\ f_3(\lambda)\\ 0\\ 0\\ 0\end{pmatrix} \tag{34}$$

$$\left.\begin{array}{l}
m_{11}(\lambda)=\left(2\tilde{\mu}+\tilde{\lambda}\right)v_\ell^2-\tilde{\lambda}\lambda^2,\quad m_{12}(\lambda)=\left[\left(2\tilde{\mu}+\tilde{\lambda}\right)v_\ell^2-\tilde{\lambda}\lambda^2\right]\mathrm{e}^{-v_\ell h_2}\\
m_{13}(\lambda)=\left(2\tilde{\mu}+\tilde{\lambda}\right)\left(-v_t^3-v_t k_t^2\right)+\tilde{\lambda}\lambda^2 v_t\\
m_{14}(\lambda)=\left[\left(2\tilde{\mu}+\tilde{\lambda}\right)\left(v_t^3+v_t k_t^2\right)-\tilde{\lambda}\lambda^2 v_t\right]\mathrm{e}^{-v_t h_2}\\
m_{21}(\lambda)=-2v_\ell,\quad m_{22}(\lambda)=2v_\ell \mathrm{e}^{-v_\ell h_2}\\
m_{23}(\lambda)=2v_t^2+k_t^2,\quad m_{24}(\lambda)=\left[2v_t^2+k_t^2\right]\mathrm{e}^{-v_t h_2}\\
m_{31}(\lambda)=-v_\ell,\quad m_{32}(\lambda)=v_\ell \mathrm{e}^{-v_\ell h_2},\quad m_{33}(\lambda)=v_t^2+k_t^2\\
m_{34}(\lambda)=\left[v_t^2+k_t^2\right]\mathrm{e}^{-v_t h_2},\quad m_{35}(\lambda)=-\omega^{-2}\tilde{\rho}_0^{-1}v_0\\
m_{41}(\lambda)=\left[\left(2\tilde{\mu}+\tilde{\lambda}\right)v_\ell^2-\tilde{\lambda}\lambda^2\right]\mathrm{e}^{-v_\ell h_2},\quad m_{42}(\lambda)=\left(2\tilde{\mu}+\tilde{\lambda}\right)v_\ell^2-\tilde{\lambda}\lambda^2\\
m_{43}(\lambda)=\left[\left(2\tilde{\mu}+\tilde{\lambda}\right)\left(-v_t^3-v_t k_t^2\right)+\tilde{\lambda}\lambda^2 v_t\right]\mathrm{e}^{-v_t h_2}\\
m_{44}(\lambda)=\left(2\tilde{\mu}+\tilde{\lambda}\right)\left(v_t^3+v_t k_t^2\right)-\tilde{\lambda}\lambda^2 v_t\\
m_{51}(\lambda)=-2v_\ell \mathrm{e}^{-v_\ell h_2},\quad m_{52}(\lambda)=2v_\ell\\
m_{53}(\lambda)=\left[2v_t^2+k_t^2\right]\mathrm{e}^{-v_t h_2},\quad m_{54}(\lambda)=2v_t^2+k_t^2\\
m_{61}(\lambda)=-v_\ell \mathrm{e}^{-v_\ell h_2},\quad m_{62}(\lambda)=v_\ell\\
m_{63}(\lambda)=\left[v_t^2+k_t^2\right]\mathrm{e}^{-v_t h_2},\quad m_{64}(\lambda)=v_t^2+k_t^2\\
m_{66}(\lambda)=\omega^{-2}\tilde{\rho}_1^{-1}v_1,\quad f_1(\lambda)=-\mathrm{e}^{-v_0 h_1},\quad f_3(\lambda)=-\omega^{-2}\tilde{\rho}_1^{-1}v_1 \mathrm{e}^{-v_0 h_1}
\end{array}\right\} \tag{35}$$

Solving the system (36), we find the function:

$$y(\lambda)=\left|M_5(\lambda)\right|Z(\lambda)/\left|M(\lambda)\right| \tag{36}$$

where $\left|M(\lambda)\right|$ is the determinant of the matrix $M(\lambda)$, $\left|M_5(\lambda)\right|$ is the determinant of the matrix $M_5(\lambda)$, $M_5(\lambda)$ is the matrix $M(\lambda)$ in which the fifth column is replaced by the vector $F(\lambda)$.

Relation between coefficients $\tilde{T}_k$ and $\tilde{X}_p$ based on the eqs. (22), (25), (28), (32) and (36) take:

$$\tilde{T}_k=\sum_{p=0}^{\infty}S_{pk}\tilde{X}_p+\tilde{\tilde{f}}_k,\ \ k=0,1,2\ \ldots \tag{37}$$

where

$$S_{pk} = 4\xi_0^3(-1)^k i^{k+p} \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_p(\xi_0) \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_k(\xi_0) \int_0^\infty \frac{|M_5(\lambda)|}{k_0 v_0 |M(\lambda)|} \times$$
$$\times P_p\left(\frac{iv_0}{k_0}\right) P_k\left(\frac{iv_0}{k_0}\right) \mathrm{e}^{-v_0 h_1} \lambda d\lambda \qquad (38)$$

$$\tilde{\tilde{f}}_k = 4\xi_0^3\, i(-i)^{k+1} \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_k(\xi_0) \int_0^\infty \frac{|M_5(\lambda)|}{v_0 |M(\lambda)|} P_k\left(\frac{iv_0}{k_0}\right) \mathrm{e}^{-v_0 h_1} \lambda d\lambda \qquad (39)$$

After excluding coefficients $\tilde{T}_k$ from the right-hand side of the eq. (29) with the help of eq. (37), then we have:

$$\tilde{X}_n - \sum_{k=0}^\infty (g_k R_{nk} - \alpha_{nk}) \tilde{X}_k = \sum_{k=0}^\infty \left(\tilde{f}_k + (-1)^k \tilde{\tilde{f}}_k \right) R_{nk}, \quad n = 0, 1, 2, \ldots \qquad (40)$$

$$\alpha_{nk} = \sum_{p=0}^\infty (-1)^p R_{np} S_{kp} \qquad (41)$$

Let's find connection between the function $d(\lambda)$, entering into representation of pressure $p_1(\rho, z)$ in area D_1, and coefficients $\tilde{X}_n$ – solutions of system (40). From eq. (33) it follows that:

$$d(\lambda) = |M_6(\lambda)| Z(\lambda) / |M(\lambda)| \qquad (42)$$

where $|M_6(\lambda)|$ is the determinant of the matrix $M_6(\lambda)$, $M_6(\lambda)$ is the matrix $M(\lambda)$, in which the sixth column is replaced by the vector $F(\lambda)$.

According to eqs. (25) and (32), we have:

$$d(\lambda) = \frac{|M_6(\lambda)|}{|M(\lambda)| k_0 v_0} \sum_{p=0}^\infty i^{-p-1} P_p\left(\frac{iv_0}{k_0}\right) \left((-1)^p \tilde{X}_p \frac{\mathrm{d}}{\mathrm{d}\xi_0} j_p(\xi_0) + f_p \right) \qquad (43)$$

The coefficient of screening of the sound field in area D_1 is calculated based on the following formula:

$$K(\rho, z) = |p_1(\rho, z)| / |p_c|, \quad z > h_1 + h_2 \qquad (44)$$

4. Computational experiment

Using computer algebra system MathCAD [17, 18], calculations of the coefficient of screening of the sound field were carried out in area D_1 for some parameters of the problem. Spherical functions were calculated by means of built-in functions. Derivatives of spherical functions were calculated by means of the formula [13]:

$$\frac{\mathrm{d}}{\mathrm{d}x} f_n(x) = nf_n(x)/x - f_{n+1}(x), \quad n = 0, 1, 2, ... \tag{45}$$

Values of $v_j = \sqrt{\lambda^2 - k_j{}^2}$, $j=0,1$, $v_\ell = \sqrt{\lambda^2 - k_\ell^2}$, $v_t = \sqrt{\lambda^2 - k_t^2}$ were calculated according to the formulae:

$$v_\tau = \begin{cases} \sqrt{\lambda^2 - k_\tau{}^2}, & \lambda \geq k_\tau \\ -i\sqrt{k_\tau{}^2 - \lambda}, & 0 \leq \lambda < k_\tau \end{cases} \tag{46}$$

The infinite system (36) was solved by the method of truncation [17]. Computational experiment showed that the truncation order for the considered parameters of a task can be eq. (25). It provides the decision of eq. (36) with an accuracy 10^{-4}. Lame coefficients are associated with the Young's modulus E and Poisson's ratio by the relation:

$$\tilde{\lambda} = \nu E/((1+\nu)(1-2\nu)), \quad \tilde{\mu} = E/(2+2\nu) \tag{47}$$

Computational experiment showed that the truncation order of the eq. (42) can be eq. (17) for the considered parameters of the problem. This provides the ultimate solution of the eq. (42) with accuracy and the condition number that will not exceed 35. Figure 2. shows plots of shielding coefficient $K(0,z)$ of the sound field $z > h_1 + h_2$, for some values of the angle θ_0. The area D_0 is filled by the air (ρ_0 = 1.29 kg/m^3, c_0 = 343 m/s). The area D_1 is filled by the water ($\tilde{\rho}_1$ =1000 kg/m^3, c_1 = 1500 m/s). The area D_1 is filled by the rubber ($\tilde{\rho}$ = 910 kg/m^3, E = 7.9 MPa, ν = 0.46). The remain parameters are equal: h_1 = 4 m; h_2 = 0.02 m; a = 0.2 m, f = 50 Hz; $\omega = 2\pi f$.

Figure 3. shows plots of shielding coefficient $K(0, z)$ of the sound field, $z > h_1 + h_2$, for some values of the frequency of the sound field. The area D_0 is filled by the air (ρ_0 = 1.29 kg/m^3, c_0 = 343 m/s). The area D_1 is filled by the nitrogen (ρ_1 = 830 kg/m^3, c_1 = 962 m/s). The area D_1 is filled by the aluminum (ρ = 2600 kg/m^3, E = 65 GPa, ν = 0.32). Remain parameters are equal: h_1 = 4 m; h_2 = 0.02 m; a = 0.3 m, $\theta_0 = \pi/2$.

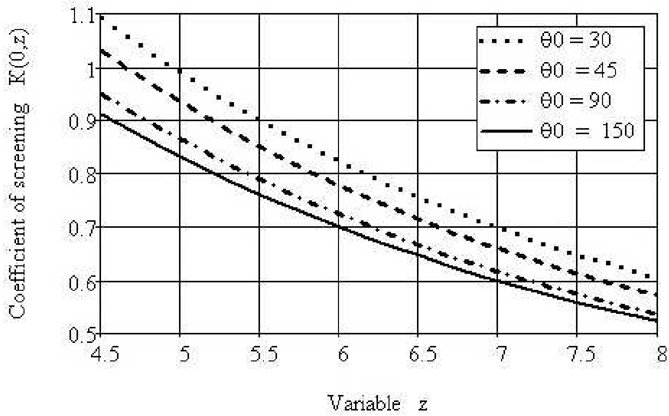

Fig. 2. Value of shielding coefficient $K(0, z)$ of the sound field for some values of the angle θ_0

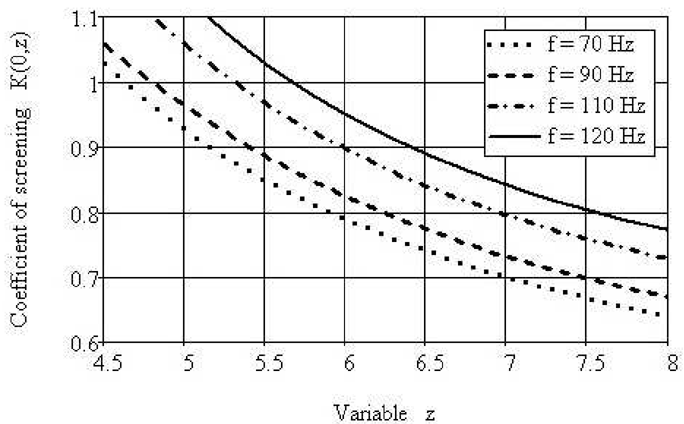

Fig. 3. Value of shielding coefficient $K(0, z)$ of the sound field for some values of the frequency f of the sound field

5. Conclusions

The solution of the problem of the penetration of the sound field through a flat elastic layer is reduced to solve dual equations in Legendre's polynomials using the addition theorem for cylindrical and spherical wave functions. The developed methodology and software can be of practical use in the manufacture of sound screens. Following tasks were carried out:

1. Dual equations are converted to the infinite system of linear algebraic equations of the second kind with the completely continuous operator.

2. The spherical radiator is considered as the source of the sound field located within the thin open spherical shell.
3. The influence of geometrical parameters of the problem, the density of the environments, Young's modulus, Poisson's ratio and the speed of sound on the value of the shielding coefficient of the sound field were computed.

Acknowledgement

The research leading to these results has received funding from the People Programme (Marie Curie International Research Staff Exchange) of the European Union's Seventh Framework Programme FP7/2007-2013/under REA grant agreement No. PIRSES-GA-2013-610547.

References

[1] Raichel D.R.: The science and applications of acoustics. Springer Science+Business Media, New York 2006.
[2] Nowacki W.: Teoria sprężystości. PWN, Warszawa 1970.
[3] Grinchenko V.T., Vovk I.V., Matsipura V.T.: Fundamentals of acoustics. Naukova dumka, Kiev 2007.
[4] Sadler J., Maev R. G.: A ray technique to calculate reflected and transmitted waves in layered media. Ultrasonics, 48 (2008), 687-696.
[5] Piquette J.C.: Interactions of a spherical wave a bilaminar plate composed of homogeneous and isotropic solid layers. J. Acoustical Soc. America, 84 (1988), 1526-1535.
[6] Gortinskaja L.V., Popov I.U.: Wave scattering in layers with hard edges, connected through the small holes. Acoustic J., 53 (2007), 485-489.
[7] Yan G., Zhao H.: The far field operator for a multilayered scatterer, J. Comp. Math. Appl., 43 (2002), 631-639.
[8] Vashishth A.K., Gupta V.: Ultrasonic wave's interaction at fluid-porous piezoelectric layered interface. J. Ultrasonics, 53 (2013), 479-494.
[9] Kiselyova N.N., Shushkevich G.C.: Computer simulation of acoustic fields in the problem shielding by the system permeable planes. [In:] Systems in teaching and research, Collegium Mazovia, Siedlce 2003, 160-169.
[10] Cheeke J.D.N.: Fundamentals and applications of ultrasonic waves reflection and transmission of ultrasonic waves at interfaces. CRC Press, New York 2002, 115-141.
[11] Maev R.G.: Acoustic microscopy: fundamentals and applications. Wiley-VCH, Wainheim 2008.
[12] Ivanov E.A.: Diffraction of electromagnetic wa.ves on two bodies. Springfield, Washington 1970.
[13] Handbook of Mathematical Functions: with Formulas, Graphs and Mathematical Tables, eds. M. Abramowitz, I.A. Stegun, Dover, New York 1972.
[14] Erofeenko V.T.: An addition theorems. Nauka and Technika, Minsk 1989.
[15] Shushkevich G.Ch., Kiselyova N.N.: Penetration of a sound field through a multilayered spherical shell. Informatika, 3 (2013), 47-57.

[16] Achenbach J.: Wave propagation in elastic solids, North-Holland Publishing, Amsterdam–New York 1973.
[17] Shushkevich G.Ch., Shushkevich S.V.: Computer technology in mathematics. The system Mathcad 14: in 2 parts, Grevsova, Minsk 2012.
[18] Fausett L.V.: Numerical Methods Using MathCAD. Prentice Hall, New York 2002.

PRZENIKANIE POLA AKUSTYCZNEGO PROMIENNIKA KULISTEGO PRZEZ PŁASKĄ WARSTWĘ SPRĘŻYSTĄ

Streszczenie

W artykule przedstawiono wyniki dokładnych obliczeń osiowosymetrycznego problemu przenikania pola akustycznego przez płaską warstwę sprężystą. Kulisty promiennik jest umieszczony w cienkiej otwartej powłoce, będącej źródłem pola akustycznego. Wykorzystując odpowiednie twierdzenia, rozwiązanie problemu warunków brzegowych ograniczono do rozwiązania podwójnych funkcji w wielomianach Legendre'a, które są transponowane do skończonych liniowych równań algebraicznych drugiego rzędu z całkowicie ciągłym operatorem. Badano wpływ niektórych parametrów problemu na wartość współczynnika ekranowania pola akustycznego.

Słowa kluczowe: warstwa sprężysta, pole akustyczne, promiennik kulisty

DOI: 10.7862/rm.2014.62

Otrzymano/received: 20.07.2014 r.
Zaakceptowano/accepted: 22.11.2014 r.