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THE PENETRATION OF THE SOUND FIELD
OF THE SPHERICAL RADIATOR THROUGH
THE PLANE ELASTIC LAYER

In this paper the results of exact solution of the axisymmetric problem of the penetration of the sound field through the plane elastic layer are presented. The spherical radiator is located in a thin unclosed spherical shell as the source of the acoustic field. Using appropriate theorems, the solution of the boundary conditions problem is reduced to solve dual functions in Legendre's polynomials, which are converted to the infinite system of linear algebraic equations of the second kind with a completely continuous operator. The influence of some parameters of the problem on the value of the coefficient of shielding sound field is investigated.

\textbf{Keywords:} elastic plate, sound field, spherical radiator

1. Introduction

The research of the distribution of the sound waves in elastic environment has a great importance in medical diagnostics, in the underwater acoustics and in the seismology, etc. \cite{1-3}. Sandler and Maev \cite{4} considered the problem of calculating the propagation of acoustic waves within an ideal isotropic multilayer plate structure. Exploring this problem by examining the ray paths of the multiple reflections within the plate structure, it is possible to show that upon careful consideration many of these paths will travel equivalent distances in time and space becoming coincident. The solution of a problem of dispersion of a spherical sound wave on a multilayered uniform firm plate can be consolidated to system of the algebraic equations \cite{5}. Results of research of distribution of sound waves in poorly connected acoustic layers with rigid borders are presented by Gortinskaja and Popov \cite{6}. For the solution of the Helmholtz equation with Neumann’s boundary conditions the method of coordination of asymptotic decomposition of solutions of regional problems is used.

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Yan and Zhao [7] considered the inverse problem of the scattering of a plane acoustic wave by a multilayered scatterer. The inverse scattering problem is analysed as the problem of determining the shape of a multilayered scatterer by measurements of the far field patterns of acoustic or electromagnetic scattered waves. Transfer matrix technique is used by Vashisht and Gupta [8] to study the layered materials. The effects of frequency, porosity, angle of incidence, layer thickness and number of layers on the energy ratios and surface impedance are studied for different configurations of the layered materials. In recent study Kiselyova and Shushkevich [9] considered the solution of a problem on penetration of a sound field through of system permeable planes. As a source of a field the spherical radiator located in a thin not closed spherical cover is considered. The layers of the plate are made up of ideal acoustic materials (linear, homogeneous, isotropic, non dispersive) with known material parameters, and the plates are assumed to be bonded such that the interfaces follow the perfectly bonded boundary conditions [10, 11].

The aim of the paper is construct the exact solution of the axisymmetric problem of the penetration of the sound field through the flat elastic layer. The influence of some parameters of the problem for the value of the coefficient of shielding sound field is investigated.

2. Problem formulation

Let all the space $\mathbb{R}^3$ be spitted by planes $S_0 (z = h_1)$ and $S_1 (z = h_1 + h_2)$ on the fields $D_0 (z < h_1)$, $D_2 (h_1 < z < h_1 + h_2)$, $D_1 (z > h_1 + h_2)$ (fig. 1.). The area $D_0$ has thin unclosed spherical shell $\Gamma_1$ perfectly, located on the sphere $\Gamma$ of the radius $a$ with the center at the point $O$. We denoted $D_0 (0) (0 \leq r < a)$ the area of space bounded by the sphere $\Gamma$ and $D_0 = D_0 (0) \bigcup \Gamma \bigcup D_0 (1)$. The distance between points $O$ and $O_1$ is equal $h_1$, $h_2$ is the distance between planes $S_0$ and $S_1$.

The point radiator of the sound waves oscillating with angular frequency $\omega$ is located at the point $O$. Areas $D_j$, $j = 0, 1$, filled with a material in which shear waves do not propagate $A$ density of the medium and a speed of sound in the area $D_j$ are denoted by $\hat{\rho}_j$, $c_j$, respectively. The area $D_2$ is a plane elastic layer. The elastic layer oscillates under the influence of the sound field. Its deformation is determined by the displacement vector $\bar{u}$ that satisfies the Lame equation [2, 3]:

$$\ddot{\bar{u}} \Delta \bar{u} + (\ddot{\lambda} + \ddot{\mu}) \text{grad} \text{div} \bar{u} + \omega^2 \hat{\rho} \bar{u} = 0$$  \hspace{1cm} (1)
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where \( \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \) is the Laplace operator, \( \lambda, \mu \) are Lame coefficients, \( \rho \) is density of the medium.

\[ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]

To solve this problem we connected spherical coordinates \( \{ r, \theta, \varphi \} \) and cylindrical coordinates \( \{ \rho, \varphi, z \} \) with the point O. The spherical shell \( \Gamma_1 \) and the plane \( S_j, j = 0, 1 \) are described as follows:

\[ \Gamma_1 = \{ r = a, \theta_0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi \} \]  \hspace{1cm} (2)
\[ S_0 = \{ z = h_1, 0 \leq \rho \leq \infty, 0 \leq \varphi \leq 2\pi \} \]  \hspace{1cm} (3)
\[ S_1 = \{ z = h_1 + h_2, 0 \leq \rho \leq \infty, 0 \leq \varphi \leq 2\pi \} \]  \hspace{1cm} (4)

Let \( p_c \) be the pressure of the sound field of the primary point radiator, \( p_0^{(0)} \) be the secondary sound pressure field in the area \( D_0^{(0)} \), \( p_0 = p_0^{(1)} + p_0^{(2)} \) be the secondary sound pressure field in the area \( D_0^{(1)} \) and \( p_1 \) be the secondary sound pressure field in the area \( D_1 \). The actual displacement and the sound pressure are calculated by the formulas \( \vec{U} = \text{Re}\left(\vec{u} e^{-i\omega t}\right) \) and \( P_j = \text{Re}\left( p_j e^{-i\omega t}\right) \). \( P_j \) is imaginary unit. The pressures of secondary sound field \( p_0^{(j)} (j = 0, 1, 2) \) and \( p_1 \) satisfies the Helmholtz equation [2, 3]:

\[ \Delta U + \omega^2 \rho U = 0 \]
\[ \Delta p_0^{(j)} + k_0^2 p_0^{(j)} = 0 \quad \text{in} \quad D_0, \quad \Delta p_1 + k_1^2 p_1 = 0 \quad \text{in} \quad D_1 \] (5)

where: \( k_0 = \omega / c_0, \quad k_1 = \omega / c_1 \) are wave numbers.

The displacement vector is determined by the formula [2]:

\[ \ddot{u} = \text{grad} \psi + \text{rot} \left( -\frac{\partial \Phi}{\partial \rho} \hat{e}_\phi \right) \] (6)

The equation (6) is satisfied in the case of propagation of small disturbances in an elastic body for steady-state motion of the particles of the body. Functions \( \psi \) and \( \Phi \) satisfy the Helmholtz equation and are defined as:

\[
\begin{align*}
\Delta \psi + k_i^2 \psi &= 0, \quad k_i = \omega / c_i, \quad c_i = \sqrt{(\lambda + 2\mu) / \rho} \\
\Delta \Phi + k_i^2 \Phi &= 0, \quad k_i = \omega / c_i, \quad c_i = \sqrt{\mu / \rho}
\end{align*}
\] (7)

where: \( \tilde{n}_i, \ c_i \) are velocity of longitudinal and transverse elastic waves respectively.

In cylindrical coordinate system components of the displacement vector are associated with the functions \( \psi \) and \( \Phi \) by relations:

\[ u_\rho = \frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \Phi}{\partial \rho \partial z}, \quad u_z = \frac{\partial \psi}{\partial z} + \frac{\partial^2 \Phi}{\partial z^2} + k_i^2 \Phi \] (8)

The solution of the diffraction problem is reduced to find the displacement vector \( \ddot{u}(u_\rho, u_z, 0) \), the pressures of the sound field \( p_0^{(j)} (j = 0, 1, 2) \) \( p_1 \) which satisfy the boundary condition on the surface of the spherical shell (acoustically hard shell):

\[ \frac{\partial}{\partial r} \left( p_c + p_0^{(0)} \right) \bigg|_{r_1} = 0 \] (9)

boundary conditions of the interaction of the sound field with an elastic layer on a plane \( S_j \):

\[ u_z \bigg|_{S_j} = \omega^2 \tilde{\rho}_j^{-1} \frac{\partial p_j}{\partial z} \bigg|_{S_j}, \quad \frac{\partial u_\rho}{\partial z} + \frac{\partial u_z}{\partial \rho} \bigg|_{S_j} = 0 \] (10)
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\[ (2\tilde{\mu} + \tilde{\lambda}) \frac{\partial u_z}{\partial z} + \tilde{\lambda} \left( \frac{u_\rho}{\rho} + \frac{\partial u_\rho}{\partial \rho} \right) \bigg|_{S_j} = -p_j \bigg|_{S_j} \]  \tag{11}

The condition at infinity \([2, 3, 12]\) can be written as:

\[ \lim_{M \to \infty} r \left( \frac{\partial p_j(M)}{\partial r} - ik_j p_j(M) \right) = 0, \quad j = 0, 1 \]  \tag{12}

where \(M\) is an arbitrary point at the space.

Condition at continuity of pressure on the open part of the spherical shell \(\Gamma/\Gamma_1\) is given by:

\[ \left( p_c + p_0^{(0)} \right) \bigg|_{\Gamma \cap \Gamma_1} = \left( p_0^{(1)} + p_0^{(2)} \right) \bigg|_{\Gamma \cap \Gamma_1} \]  \tag{13}

and the normal derivative on the surface of the sphere \(\Gamma\) is:

\[ \frac{\partial}{\partial r} \left( p_c + p_0^{(0)} \right) \bigg|_{\Gamma} = \frac{\partial}{\partial r} \left( p_0^{(1)} + p_0^{(2)} \right) \bigg|_{\Gamma} \]  \tag{14}

The initial pressure of the sound field can be represented in the form \([12]\):

\[ p_c(r, \theta) = P \exp(ik_0r) / r = P \sum_{n=0}^{\infty} f_n h_n^{(1)}(k_0r) P_n(\cos \theta), \quad f_n = ik_0 \delta_{0n} \]  \tag{15}

where \(h_n^{(1)}(x)\) are spherical Hankel functions, \(P_n(\cos \theta)\) is Legendre polynomials \([13]\), \(\delta_{0n}\) is Kronecker delta and \(P\) is constant.

The pressure of the scattered sound field is represented as a superposition of basic solutions of the Helmholtz equation in spherical and cylindrical coordinates \([14]\), taking into account the condition at infinity (12) we have:

\[ p_0^{(0)}(r, \theta) = P \sum_{n=0}^{\infty} c_n j_n(k_0r) P_n(\cos \theta) \quad \text{in} \quad D_0^{(0)} \]  \tag{16}

\[ \begin{align*}
\sum_{n=0}^{\infty} \frac{x_n - f_n}{d_{\xi_0}} P_n(\cos \theta) &= 0, \quad 0 \leq \theta < \theta_0 \\
\sum_{n=0}^{\infty} x_n \frac{d}{d_{\xi_0}} h_n^{(1)}(\xi_0) P_n(\cos \theta) &= -\sum_{n=0}^{\infty} T_n \frac{d}{d_{\xi_0}} j_n(\xi_0) P_n(\cos \theta), \quad \theta_0 < \theta \leq \pi
\end{align*} \]  \tag{17}
\[ p_1(\rho, z) = P \int_0^\infty d(\lambda) J_0(\lambda \rho) e^{-\nu z} J_0(\lambda h) \lambda d\lambda \quad \text{in} \quad D_1 \] (18)

\[ \psi(\rho, z) = P \int_0^\infty \left( a(\lambda) e^{-\nu z} + b(\lambda) e^{\nu z} \right) J_0(\lambda \rho) \lambda d\lambda \] (19)

\[ \Phi(\rho, z) = P \int_0^\infty \left( \tilde{a}(\lambda) e^{-\nu z} + \tilde{b}(\lambda) e^{\nu z} \right) J_0(\lambda \rho) \lambda d\lambda \] (20)

where \( j_n(x) \) are spherical Bessel functions of the first kind, \( J_0(x) \) are Bessel functions of the first kind, \( \nu_j = \sqrt{\lambda^2 - k_j^2}, \quad -\pi/2 \leq \arg \nu_j < \pi/2, \quad j = 0, 1; \)

\( \nu_\epsilon = \sqrt{\lambda^2 - k_\epsilon^2}, \quad -\pi/2 \leq \arg \nu_\epsilon < \pi/2, \quad \nu_i = \sqrt{\lambda^2 - k_i^2}, \quad -\pi/2 \leq \arg \nu_i < \pi/2. \)

Unknown coefficients \( c_n, x_n \) and functions \( a(\lambda), b(\lambda), \tilde{a}(\lambda), \tilde{b}(\lambda), y(\lambda), d(\lambda) \) must be determined from boundary conditions.

3. Boundary conditions

The boundary conditions are defined by eqs. (1), (9) and (11). The function \( p_0^{(2)}(\rho, z) \) through spherical wave functions, using the formula connecting cylindrical and spherical wave functions is:

\[ J_0(\lambda \rho) e^{\nu z} = \sum_{n=0}^{\infty} (-i)^n (2n+1) P_n \left( \frac{i \nu}{k} \right) j_n(\lambda r) P_n(\cos \theta) \] (21)

then

\[ p_0^{(2)}(r, \theta) = \frac{\lambda}{k} \sum_{n=0}^{\infty} T_n j_n(k_0 r) P_n(\cos \theta) \] (22)

\[ T_n = (-i)^n (2n+1) \int_0^\infty y(\lambda) P_n \left( \frac{i \nu_0}{k_0} \right) e^{-\nu_0 \lambda} \lambda d\lambda \]

According to eqs. (12)-(14) and eq. (17), the boundary condition (11) taking into account the condition of orthogonality of Legendre polynomials on the interval \([0; \pi]\) will become:
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\[
\begin{align*}
  & f_n \frac{d}{d \xi_0} h_n^{(1)} (\xi_0) + c_n \frac{d}{d \xi_0} j_n (\xi_0) = x_n \frac{d}{d \xi_0} h_n^{(1)} (\xi_0) + T_n \frac{d}{d \xi_0} j_n (\xi_0), \\
  & \xi_0 = k_0 a, \quad n = 0, 1, \ldots
\end{align*}
\]  

(23)

Let’s specify the boundary condition (9) on the surface of a spherical shell and the condition of continuity (13). Let’s exclude factors \( c_n \) in the resulting equations, using the eq. (23). Then dual equations in Legendre's polynomials take the form:

\[
\begin{align*}
  & \sum_{n=0}^{\infty} \frac{x_n - f_n}{d \xi_0} P_n (\cos \theta) = 0, \quad 0 \leq \theta < \theta_0 \\
  & \sum_{n=0}^{\infty} x_n \frac{d}{d \xi_0} h_n^{(1)} (\xi_0) P_n (\cos \theta) = - \sum_{n=0}^{\infty} T_n \frac{d}{d \xi_0} j_n (\xi_0) P_n (\cos \theta), \quad \theta_0 < \theta \leq \pi
\end{align*}
\]

(24)

Let a new coefficient be:

\[
x_n = X_n \frac{d}{d \xi_0} j_n (\xi_0) + f_n, \quad n = 0, 1, \ldots
\]

(25)

and a small parameter be:

\[
g_n = 1 + \frac{4i \varepsilon_0^3}{2n + 1} \frac{d}{d \xi_0} j_n (\xi_0) \frac{d}{d \xi_0} h_n^{(1)} (\xi_0)
\]

(26)

Then we will make replacement \( \theta = \pi - \tilde{\theta}, \quad \theta_0 = \pi - \tilde{\theta}_0, \quad \tilde{X}_n = (-1)^n X_n \) for the transformation of dual eqs. (19). As a result, dual eqs. (24) take the form:

\[
\begin{align*}
  & \sum_{n=0}^{\infty} (2n+1)(1 - g_n) \tilde{X}_n P_n (\cos \tilde{\theta}) = \sum_{n=0}^{\infty} (-1)^n (2n+1)(\tilde{f}_n + \tilde{T}_n) P_n (\cos \tilde{\theta}), \quad 0 \leq \tilde{\theta} < \tilde{\theta}_0 \\
  & \sum_{n=0}^{\infty} \tilde{X}_n P_n (\cos \tilde{\theta}) = 0, \quad \tilde{\theta}_0 < \tilde{\theta} \leq \pi
\end{align*}
\]

(27)

where

\[
\tilde{T}_n = 4i \varepsilon_0^3 T_n \frac{d}{d \xi_0} j_n (\xi_0) / (2n+1), \quad \tilde{f}_n = 4i \varepsilon_0^3 f_n \frac{d}{d \xi_0} h_n^{(1)} (\xi_0) / (2n+1)
\]

(28)
Dual eqs. (25) are converted to an infinite system of linear algebraic equations of the second kind with the completely continuous operator using the integral representation for the Legendre polynomials [15, 16]:

\[
\tilde{X}_n - \sum_{k=0}^{\infty} g_k R_{nk} \tilde{X}_k = \sum_{k=0}^{\infty} (-1)^k \left( \tilde{T}_k + \tilde{f}_k \right) R_{nk}, \quad n = 0, 1, \ldots
\]  

(29)

where

\[
R_{nk} = \frac{1}{\pi} \left[ \sin(n-k)(\pi-\theta_0) - \frac{\sin(n+k+1)(\pi-\theta_0)}{n+k+1} \right]
\]

\[
\sin(n-k)(\pi-\theta_0) \bigg|_{n=k} = \pi - \theta_0
\]

(30)

To satisfy boundary conditions (11), the function \( p_0^{(1)}(r, \theta) \) through cylindrical wave functions takes the form:

\[
h_n^{(1)}(kr) P_n(\cos \theta) = \int_0^\infty \frac{i^{-n-1}}{kv} P_n \left( \frac{i_v \lambda}{k} \right) J_0(\lambda \rho) e^{-\nu \rho} \lambda d\lambda
\]

\[
v = \sqrt{\lambda^2 - k^2}, \quad -\pi/2 \leq \arg v < \pi/2, \quad z > 0
\]

(31)

then

\[
p_0^{(1)}(\rho, z) = P_0^\infty Z(\lambda) J_0(\lambda \rho) e^{-\nu \rho} \lambda d\lambda, \quad Z(\lambda) = \frac{1}{k_0 v_0} \sum_{n=0}^{\infty} i^{-n-1} P_n \left( \frac{iv_0}{k_0} \right) x_n
\]

(32)

Taking into account the eqs. (17)-(20) and (32) and boundary conditions (11), the linear algebraic equation takes the form:

\[
M(\lambda) \cdot V(\lambda) = F(\lambda) \cdot Z(\lambda)
\]

(33)

where

\[
M(\lambda) = \begin{pmatrix}
m_{11}(\lambda) & m_{12}(\lambda) & m_{13}(\lambda) & m_{14}(\lambda) & 1 & 0 \\
m_{21}(\lambda) & m_{22}(\lambda) & m_{23}(\lambda) & m_{24}(\lambda) & 0 & 0 \\
m_{31}(\lambda) & m_{32}(\lambda) & m_{33}(\lambda) & m_{34}(\lambda) & m_{35}(\lambda) & 0 \\
m_{41}(\lambda) & m_{42}(\lambda) & m_{43}(\lambda) & m_{44}(\lambda) & 0 & 1 \\
m_{51}(\lambda) & m_{52}(\lambda) & m_{53}(\lambda) & m_{54}(\lambda) & 0 & 0 \\
m_{61}(\lambda) & m_{62}(\lambda) & m_{63}(\lambda) & m_{64}(\lambda) & 0 & m_{66}(\lambda)
\end{pmatrix}
\]
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\[
V(\lambda) = \begin{pmatrix}
a(\lambda) \\
b(\lambda) \\
\tilde{a}(\lambda) \\
\tilde{b}(\lambda) \\
y(\lambda) \\
d(\lambda)
\end{pmatrix}, \quad F(\lambda) = \begin{pmatrix}
f_1(\lambda) \\
0 \\
0 \\
f_3(\lambda) \\
0 \\
0
\end{pmatrix}
\]  

(34)

\[
m_{11}(\lambda) = (2\mu + \tilde{\lambda})v_i^2 - \tilde{\lambda}\lambda^2, \quad m_{12}(\lambda) = \left[ (2\mu + \tilde{\lambda})v_i^2 - \tilde{\lambda}\lambda^2 \right] e^{-\nu_i h_i}
\]

\[
m_{13}(\lambda) = (2\mu + \tilde{\lambda})(-v_i^3 - v_i k_i^2) + \tilde{\lambda}\lambda^2 v_i
\]

\[
m_{14}(\lambda) = \left[ (2\mu + \tilde{\lambda})(v_i^3 + v_i k_i^2) - \tilde{\lambda}\lambda^2 v_i \right] e^{-\nu_i h_i}
\]

\[
m_{21}(\lambda) = -2v_i, \quad m_{22}(\lambda) = 2v_i e^{-\nu_i h_i}
\]

\[
m_{23}(\lambda) = 2v_i^2 + k_i^2, \quad m_{24}(\lambda) = \left[ 2v_i^2 + k_i^2 \right] e^{-\nu_i h_i}
\]

\[
m_{31}(\lambda) = -v_i, \quad m_{32}(\lambda) = v_i e^{-\nu_i h_i}, \quad m_{33}(\lambda) = v_i^2 + k_i^2
\]

\[
m_{34}(\lambda) = \left[ v_i^2 + k_i^2 \right] e^{-\nu_i h_i}, \quad m_{35}(\lambda) = -\omega^2 \tilde{\rho}_i^{-1} v_i
\]

\[
m_{41}(\lambda) = \left[ (2\mu + \tilde{\lambda})v_i^2 - \tilde{\lambda}\lambda^2 \right] e^{-\nu_i h_i}, \quad m_{42}(\lambda) = (2\mu + \tilde{\lambda})v_i^2 - \tilde{\lambda}\lambda^2
\]

\[
m_{43}(\lambda) = \left[ (2\mu + \tilde{\lambda})(-v_i^3 - v_i k_i^2) + \tilde{\lambda}\lambda^2 v_i \right] e^{-\nu_i h_i}
\]

\[
m_{44}(\lambda) = (2\mu + \tilde{\lambda})(v_i^3 + v_i k_i^2) - \tilde{\lambda}\lambda^2 v_i
\]

\[
m_{51}(\lambda) = -2v_i e^{-\nu_i h_i}, \quad m_{52}(\lambda) = 2v_i e^{-\nu_i h_i}
\]

\[
m_{53}(\lambda) = 2v_i^2 + k_i^2, \quad m_{54}(\lambda) = 2v_i^2 + k_i^2
\]

\[
m_{61}(\lambda) = -v_i e^{-\nu_i h_i}, \quad m_{62}(\lambda) = v_i e^{-\nu_i h_i}
\]

\[
m_{63}(\lambda) = \left[ v_i^2 + k_i^2 \right] e^{-\nu_i h_i}, \quad m_{64}(\lambda) = v_i^2 + k_i^2
\]

\[
m_{66}(\lambda) = \omega^2 \tilde{\rho}_i^{-1} v_i, \quad f_1(\lambda) = -e^{-\nu_i h_i}, \quad f_3(\lambda) = -\omega^2 \tilde{\rho}_i^{-1} v_i e^{-\nu_i h_i}
\]

Solving the system (36), we find the function:

\[
y(\lambda) = |M_5(\lambda)|Z(\lambda) / |M(\lambda)|
\]  

(36)

where \( |M(\lambda)| \) is the determinant of the matrix \( M(\lambda) \), \( |M_5(\lambda)| \) is the determinant of the matrix \( M_5(\lambda) \), \( M_5(\lambda) \) is the matrix \( M(\lambda) \) in which the fifth column is replaced by the vector \( F(\lambda) \).

Relation between coefficients \( \tilde{T}_k \) and \( \tilde{X}_p \) based on the eqs. (22), (25), (28), (32) and (36) take:

\[
\tilde{T}_k = \sum_{p=0}^{\infty} S_{pk} \tilde{X}_p + \tilde{f}_k, \quad k = 0, 1, 2 \ldots
\]  

(37)
\[ S_{pk} = 4\xi_0 (\xi_0) (-1)^k i^{k+1} \frac{d}{d\xi_0} j_p (\xi_0) \frac{d}{d\xi_0} j_k (\xi_0) \int_0^\infty \frac{|M_6(\lambda)|}{k_0 v_0 |M(\lambda)|} \times \]
\[ \times P_p \left( \frac{iv_0}{k_0} \right) P_k \left( \frac{iv_0}{k_0} \right) e^{-v_h \lambda} d\lambda \]

(38)

\[ \tilde{f}_k = 4\xi_0 i (-i)^{k+1} \frac{d}{d\xi_0} j_k (\xi_0) \int_0^\infty \frac{|M_6(\lambda)|}{v_0 |M(\lambda)|} P_k \left( \frac{iv_0}{k_0} \right) e^{-v_h \lambda} d\lambda \]

(39)

After excluding coefficients \( \tilde{T}_k \) from the right-hand side of the eq. (29) with the help of eq. (37), then we have:

\[ \tilde{X}_n - \sum_{k=0}^\infty (g_k R_{nk} - \alpha_{nk}) \tilde{X}_k = \sum_{k=0}^\infty (\tilde{f}_k + (-1)^k \tilde{f}_k) R_{nk}, \quad n = 0, 1, 2, \ldots \]  

(40)

\[ \alpha_{nk} = \sum_{p=0}^\infty (-1)^p R_{np} S_{kp} \]  

(41)

Let's find connection between the function \( d(\lambda) \), entering into representation of pressure \( p_1(\rho, z) \) in area \( D_1 \), and coefficients \( \tilde{X}_n \) – solutions of system (40). From eq. (33) it follows that:

\[ d(\lambda) = |M_6(\lambda)| Z(\lambda) / |M(\lambda)| \]  

(42)

where \( |M_6(\lambda)| \) is the determinant of the matrix \( M_6(\lambda) \), \( M_6(\lambda) \) is the matrix \( M(\lambda) \), in which the sixth column is replaced by the vector \( F(\lambda) \).

According to eqs. (25) and (32), we have:

\[ d(\lambda) = \frac{|M_6(\lambda)|}{|M(\lambda)| k_0 v_0} \sum_{p=0}^\infty (-1)^p \tilde{X}_p \frac{d}{d\xi_0} j_p (\xi_0) + f_p \]

(43)

The coefficient of screening of the sound field in area \( D_1 \) is calculated based on the following formula:

\[ K(\rho, z) = \left| p_1(\rho, z) / |p_1| \right|, \quad z > h_1 + h_2 \]  

(44)
4. Computational experiment

Using computer algebra system MathCAD [17, 18], calculations of the coefficient of screening of the sound field were carried out in area $D_1$ for some parameters of the problem. Spherical functions were calculated by means of built-in functions. Derivatives of spherical functions were calculated by means of the formula [13]:

$$\frac{d}{dx} f_n(x) = nf_n(x) / x - f_{n+1}(x), \quad n = 0, 1, 2, ...$$

(45)

Values of $v_j = \sqrt{\lambda^2 - k_j^2}$, $j = 0, 1$, $v_{t} = \sqrt{\lambda^2 - k_{t}^2}$, $v_{r} = \sqrt{\lambda^2 - k_{r}^2}$ were calculated according to the formulae:

$$v_{r} = \begin{cases} \sqrt{\lambda^2 - k_{r}^2}, & \lambda \geq k_{r} \\ -i\sqrt{k_{r}^2 - \lambda}, & 0 \leq \lambda < k_{r} \end{cases}$$

(46)

The infinite system (36) was solved by the method of truncation [17]. Computational experiment showed that the truncation order for the considered parameters of a task can be eq. (25). It provides the decision of eq. (36) with an accuracy $10^{-4}$. Lame coefficients are associated with the Young's modulus $E$ and Poisson's ratio by the relation:

$$\tilde{\lambda} = \nu E / ((1 + \nu)(1 - 2\nu)), \quad \tilde{\mu} = E / (2 + 2\nu)$$

(47)

Computational experiment showed that the truncation order of the eq. (42) can be eq. (17) for the considered parameters of the problem. This provides the ultimate solution of the eq. (42) with accuracy and the condition number that will not exceed 35. Figure 2. shows plots of shielding coefficient $K(0, z)$ of the sound field $z > h_1 + h_2$, for some values of the angle $\theta_0$. The area $D_0$ is filled by the air ($\rho_0 = 1.29$ kg/m$^3$, $c_0 = 343$ m/s). The area $D_1$ is filled by the water ($\rho_1 = 1000$ kg/m$^3$, $c_1 = 1500$ m/s). The area $D_1$ is filled by the rubber ($\rho = 910$ kg/m$^3$, $E = 7.9$ MPa, $\nu = 0.46$). The remain parameters are equal: $h_1 = 4$ m; $h_2 = 0.02$ m; $a = 0.2$ m, $f = 50$ Hz; $\omega = 2\pi f$.

Figure 3. shows plots of shielding coefficient $K(0, z)$ of the sound field, $z > h_1 + h_2$, for some values of the frequency of the sound field. The area $D_0$ is filled by the air ($\rho_0 = 1.29$ kg/m$^3$, $c_0 = 343$ m/s). The area $D_1$ is filled by the nitrogen ($\rho_1 = 830$ kg/m$^3$, $c_1 = 962$ m/s). The area $D_1$ is filled by the aluminum ($\rho = 2600$ kg/m$^3$, $E = 65$ GPa, $\nu = 0.32$). Remain parameters are equal: $h_1 = 4$ m; $h_2 = 0.02$ m; $a = 0.3$ m, $\theta_0 = \pi/2$. 
5. Conclusions

The solution of the problem of the penetration of the sound field through a flat elastic layer is reduced to solve dual equations in Legendre's polynomials using the addition theorem for cylindrical and spherical wave functions. The developed methodology and software can be of practical use in the manufacture of sound screens. Following tasks were carried out:

1. Dual equations are converted to the infinite system of linear algebraic equations of the second kind with the completely continuous operator.
2. The spherical radiator is considered as the source of the sound field located within the thin open spherical shell.
3. The influence of geometrical parameters of the problem, the density of the environments, Young's modulus, Poisson's ratio and the speed of sound on the value of the shielding coefficient of the sound field were computed.

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PRZENIKANIE POLA AKUSTYCZNEGO PROMIENNIKA KULISTEGO PRZEZ PŁASKĄ WARSTwę SPREGŻYSTą

Streszczenie

W artykule przedstawiono wyniki dokładnych obliczeń osiowoosymetrycznego problemu przenikania pola akustycznego przez płaską warstwę sprężystą. Kulisty promiennik jest umieszczony w cienkiej otwartej powłoce, będącej źródłem pola akustycznego. Wykorzystując odpowiednie twierdzenia, rozwiązanie problemu warunków brzegowych ograniczono do rozwiązania podwójnych funkcji w wielomianach Legendre’a, które są transponowane do skończonych linio-wych równań algebraicznych drugiego rzędu z całkowicie ciągłym operatorem. Badano wpływ niektórych parametrów problemu na wartość współczynnika ekranowania pola akustycznego.

Słowa kluczowe: warstwa sprężysta, pole akustyczne, promiennik kulisty

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