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PROPAGATION OF LOW-FREQUENCY ELECTROMAGNETIC WAVES ACROSS A MULTILAYER CYLINDRICAL SHELL

In the paper the results of the analysis of propagation of the low-frequency electromagnetic waves across the multilayer cylindrical shell are presented. The mathematical model of non-local bilateral boundary conditions of multilayer cylindrical shells was created. These non-local bilateral boundary conditions describe the penetration of low-frequency electromagnetic waves across layers of the shell. The method of transformation of the boundary problem to the integral Fredholm equations of the second kind was developed. Furthermore, the value of efficiency coefficient of the shell was calculated. Analytical transformations are based on the Bessel functions.

Keywords: Bessel function, boundary conditions, electromagnetic waves, electromagnetic field

1. Introduction

Controllable propagation of electromagnetic wave in a confined space which dimension is smaller than the incident wavelength is an interesting and challenging topic [1]. A new adverse environmental factor has been formed in recent years. This leads to decreasing the efficiency of hardware and staff [2, 3]. A low-frequency field affects the human especially, because biological functions are largely based on low-frequency rhythms [4]. That is why the creation of different types of screens that improves protection of technical devices, as well as biological effects of magnetic fields is important [5-7]. It is known that multi-layered screens material improves the efficiency of screening [8-12]. Depending on the frequency of external magnetic fields, electromagnetic shielding occurs

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through different physical mechanisms. For static and low-frequency fields, the most important mechanism is flux shunting due to a high magnetic permeability [13]. A number of publications were devoted to the development of an analytical approach to derive simple formulae for the shielding efficiency for different geometries: cubic and cylindrical shields of finite length and spheroidal shields [14]. Sumner et.al. [15] presented a systematic review of a theoretical treatment and comparison with practical realizations. The detail analysis of various shielding geometries and configurations is presented by Rikitake [16].

In the paper the analitycal analysis of electromagnetic waves propagation across the multilayer cylindrical shell is carried out. The aim of the paper is to develop the analitycal method of transformation of boundary conditions to the integral Fredholm equations of the second type using Bessel functions. The relation between the efficiency coefficient E and a number of layers n is also analyzed. The developed analitycal method uses average non-local bilateral boundary conditions, which describe the propagation of low-frequency electromagnetic waves across the multilayer cylindrical shell. Average non-local bilateral boundary conditions are related to the waves on either side of the shield. A similar procedure for the electromagnetic field was developed by Erofeenko et.al. [12].

2. Problem formulation

The thin-wall cylindrical shell $D\{R_1 < \rho < R_2, \ |z| < \ell, \ 0 \le \varphi < 2\pi\}$ of the thickness $h = R_2 - R_1$ and the height 2ℓ is located in the space R^3 with the permittivity ε_0 and the permeability μ_0. Vector $\{\rho, \varphi, z\}$ is cylindrical coordinates at point O (fig. 1.). The cylinder consists of $n + 1$ cylindrical layers $\Omega_s \{\rho_s < \rho < \rho_{s+1}, \ |z| < \ell, \ 0 \le \varphi < 2\pi\}$ with the permittivity $\varepsilon^{(s)}$ and the permeability $\mu^{(s)}$, $s = 0,1,...,n$. Parameters $\Delta_s = \rho_{s+1} - \rho_s$ and $h = \sum_{s=0}^{n} \Delta_s$ are the thickness of the s-layer and thickness of shell D, respectively.

We denote by $\Gamma_- \{\rho = R_1, \ |z| < \ell, 0 \le \varphi < 2\pi\}$ the inner surface and $\Gamma_+ \{\rho = R_2, \ |z| < \ell, 0 \le \varphi < 2\pi\}$ the outer surface of the shell D. We introduce the cylindrical surface $\Gamma\{\rho = R = 0,5(R_1 + R_2), \ |z| < \infty, \ 0 \le \varphi < 2\pi\}$, which divides inner $D_1\{0 \le \rho < R, \ |z| < \infty, \ 0 \le \varphi < 2\pi\}$ and outer cylindrical region $D_2\{\rho > R, \ |z| < \infty, \ 0 \le \varphi < 2\pi\}$. The surface $\Gamma = \Gamma_0 \bigcup \Gamma_c$, $\Gamma_0\{\rho = R, \ |z| > \ell, \ 0 \le \varphi < 2\pi\}$, $\Gamma_c\{\rho = R, \ |z| < \ell, \ 0 \le \varphi < 2\pi\}$ is the median surface of the shell D. The primary low-frequency magnetic field $\vec{H}_0 = -H_0 \vec{e}_x$

influences on the shell D with a circular frequency ω. The formula for magnetic potential is:

$$u_0 = H_0 \, \rho \cos\varphi, \quad H_0 - \text{const.} \tag{1}$$

Let u_1' be the magnetic potential of secondary magnetic field at D_1, u_2' be the magnetic potential of reflected magnetic field at D_2 and $u_j = u_0 + u_j'$ be the total magnetic potential magnetic field at D_j, $j = 1, 2$. Magnetic fields are defined by the formulas:

$$\vec{H}_0 = -\operatorname{grad} u_0, \quad \vec{H}_j = -\operatorname{grad} u_j, \quad j = 1, 2 \tag{2}$$

Actual magnetic fields are defined by the formulas:

$$\vec{H}_0 = -\operatorname{grad} \operatorname{Re}\left(u_0 e^{-i\omega t}\right), \quad \vec{H}_j = -\operatorname{grad} \operatorname{Re}\left(u_j e^{-i\omega t}\right) \tag{3}$$

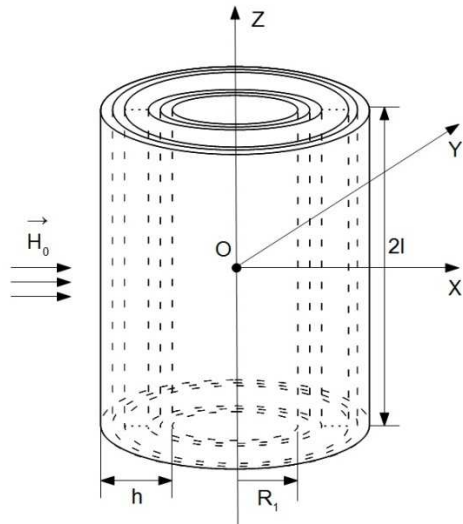where $\omega = 2\pi f$ is the circular frequency of magnetic field.

Fig. 1. Geometry of the problem

Electromagnetic fields $\vec{E}^{(s)}, \vec{H}^{(s)}$ satisfy the Maxwell equations in the layers Ω_s of the shell D:

$$\operatorname{rot} \vec{E}^{(s)} = i\omega\mu^{(s)} \vec{H}^{(s)}, \quad \operatorname{rot} \vec{H}^{(s)} = -i\omega\mu^{(s)} \vec{E}^{(s)}, \quad \mu^{(s)} = \mu_r^{(s)}\mu_0, \quad \varepsilon^{(s)} = \varepsilon_r^{(s)}\varepsilon_0 \tag{4}$$

Let us formulate the boundary problem of the magnetic field $\vec{H}_0$ and shell D interaction. We need to define potentials $u_j' \in C^2(D_j) \cap C^1(\bar{D}_j)$ for the primary potential u_0 that satisfy Laplace equation:

$$\Delta u_j' = 0 \ \text{in} \ D_j \tag{5}$$

The boundary conditions take the form:

$$u_2\big|_{\Gamma_0} = u_1\big|_{\Gamma_0}, \quad \frac{\partial u_2}{\partial \rho}\bigg|_{\Gamma_0} = \frac{\partial u_1}{\partial \rho}\bigg|_{\Gamma_0} \tag{6}$$

$$Q_1 \frac{\partial}{\partial \rho} u_1(M_1)\bigg|_{\Gamma_-} + Q_2 \frac{\partial}{\partial \rho} u_2(M_2)\bigg|_{\Gamma_+} = F\left(u_1(M_1)\big|_{\Gamma_-} - u_2(M_2)\big|_{\Gamma_+}\right) \tag{7}$$

$$\frac{\partial}{\partial \rho} u_1(M_1)\bigg|_{\Gamma_-} - \frac{\partial}{\partial \rho} u_2(M_2)\bigg|_{\Gamma_+} = F\left(P_1 u_1(M_1)\big|_{\Gamma_-} + P_2 u_2(M_2)\big|_{\Gamma_+}\right) \tag{8}$$

where

$$F(u) = \left(\vec{n}, \text{rot}[\vec{n}, \text{grad}\, u]\right) = \frac{\partial^2 u}{\partial z^2} + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2}, \quad \vec{n} = \vec{e}_\rho \tag{9}$$

$$Q_j = i\omega\mu_0 q_j, \ \ P_j = p_j / i\omega\mu_0, \ \ M_j = (\rho = R_j, \varphi, z), \ \ j = 1, 2, \ \ |z| < \ell \tag{10}$$

Finally, the infinity conditions become:

$$r u_1'(M) \to 0, \ \text{when} \ r \to \infty, \ \ r u_2'(M) \to 0 \ \text{when} \ r \to \infty \tag{11}$$

where r is the spherical coordinate of point M.

It is noticed that bilateral boundary conditions (8) simulate the magnetic field $\vec{H}_0$ penetration through D.

3. Boundary conditions

The algorithm for calculating the coefficients p_j, q_j of boundary conditions (8) is as follows. Let us consider transfer matrices for the layers Ω_s:

$$\widehat{a}_s = \begin{pmatrix} \cos(k_s\,\Delta_s) & iZ_s \sin(k_s\,\Delta_s) \\ i\sin(k_s\,\Delta_s)/Z_s & \cos(k_s\,\Delta_s) \end{pmatrix} \tag{12}$$

where $Z_s = \omega\mu_s/k_s$, $k_s = \omega\sqrt{\varepsilon^{(s)}\mu^{(s)}}$, $0 \le \arg k_s < \pi$.

We can use the matrix (12) for the transformation of electromagnetic waves that propagated across the plane layer Ω_s with a thickness Δ_s. Let us build the transfer matrix $\widehat{B}$ for the multilayer shell D, where:

$$\widehat{B} = \widehat{a}_n\,\widehat{a}_{n-1}\ldots\widehat{a}_1\widehat{a}_0 = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \tag{13}$$

Taking into account the matrix elements b_{jk}, we can define:

$$\left. \begin{array}{ll} p_1 = (b_{22}-1)\big/b_{21}, & p_2 = (b_{11}-1)\big/b_{21} \\ q_1 = (b_{11}-1)\big/b_{12}, & q_2 = (b_{22}-1)\big/b_{12} \end{array} \right\} \tag{14}$$

Let us transform non-local boundary conditions (6) and (8). Let us represent magnetic potentials of D_j in the form $u'_j = v'_j\cos\varphi$, $u_0 = v_0\cos\varphi$, $v_0 = H_0\rho$. We transfer boundary conditions (8) to the median surface Γ_c for simplify the model. Then we obtain:

$$(v'_2 - v'_1)\big|_{\Gamma_0} = 0, \quad \left(\frac{\partial v'_2}{\partial\rho} - \frac{\partial v'_1}{\partial\rho}\right)\bigg|_{\Gamma_0} = 0 \tag{15}$$

$$Q_1\frac{\partial}{\partial\rho}(v'_1+v_0)\bigg|_{\Gamma_-} + Q_2\frac{\partial}{\partial\rho}(v'_2+v_0)\bigg|_{\Gamma_+} =$$

$$= \left(\frac{\partial^2}{\partial z^2}(v'_1+v_0) - \frac{1}{\rho^2}v_0\right)\bigg|_{\Gamma_-} - \left(\frac{\partial^2}{\partial z^2}(v'_2+v_0) - \frac{1}{\rho^2}v_0\right)\bigg|_{\Gamma_+} + \frac{1}{\rho^2}(v'_2-v'_1)\bigg|_{\Gamma_c} \tag{16}$$

$$\left(\frac{\partial v'_1}{\partial\rho} - \frac{\partial v_2}{\partial\rho}\right)\bigg|_{\Gamma_c} = P_1\left(\frac{\partial^2}{\partial z^2}(v'_1+v_0) - \frac{1}{\rho^2}(v'_1+v)\right)\bigg|_{\Gamma_-} +$$

$$+ P_2\left(\frac{\partial^2}{\partial z^2}(v'_2+v_0) - \frac{1}{\rho^2}(v'_2+v_0)\right)\bigg|_{\Gamma_+} \tag{17}$$

Let us write the boundary conditions in the compact form:

$$\left(v_2' - v_1'\right)\Big|_{\Gamma_0} = 0, \quad \left(\frac{\partial v_2'}{\partial \rho} - \frac{\partial v_1'}{\partial \rho}\right)\Big|_{\Gamma_0} = 0 \tag{18}$$

$$\left(v_2' - v_1'\right)\Big|_{\Gamma_u} = f_1(z) + C_1, \quad \left(\frac{\partial v_2'}{\partial \rho} - \frac{\partial v_1'}{\partial \rho}\right)\Big|_{\Gamma_u} = f_2(z) + C_2, \quad |z| < \ell \tag{19}$$

where

$$f_1(z) = R^2\left(Q_1 \frac{\partial v_1'}{\partial \rho} - \frac{\partial^2 v_1'}{\partial z^2}\right)\Big|_{\rho = R_1} + R^2\left(Q_2 \frac{\partial v_2'}{\partial \rho} + \frac{\partial^2 v_2'}{\partial z^2}\right)\Big|_{\rho = R_2}, \quad |z| < l \tag{20}$$

$$f_2(z) = P_1\left(\frac{v_1'}{R_1^2} - \frac{\partial^2 v_1'}{\partial z^2}\right)\Big|_{\rho = R_1} + P_2\left(\frac{v_2'}{R_2^2} - \frac{\partial^2 v_2'}{\partial z^2}\right)\Big|_{\rho = R_2}, \quad |z| < l \tag{21}$$

$$C_1 = R^2\left(Q_2 \frac{\partial v_0}{\partial \rho} - \frac{v_0}{\rho^2}\right)\Big|_{\rho = R_2} + R^2\left(Q_1 \frac{\partial v_0}{\partial \rho} + \frac{1}{\rho^2} v_0\right)\Big|_{\rho = R_1} =$$

$$= H_0 R^2\left(Q_1 + Q_2 + \frac{1}{R_1 R_2}\right) \tag{22}$$

$$C_2 = -P_1\left(\frac{\partial^2 v_0}{\partial z^2} - \frac{1}{\rho^2} v_0\right)\Big|_{\rho = R_1} - P_2\left(\frac{\partial^2 v_0}{\partial z^2} - \frac{1}{\rho^2} v_0\right)\Big|_{\rho = R_2} = H_0\left(\frac{P_1}{R_1} + \frac{P_2}{R_2}\right) \tag{23}$$

The cylinder Γ_c of fins $\gamma(\pm) = \{z = \pm\ell, \rho = R, 0 \le \varphi < 2\pi\}$ must satisfy the requirements of energy limbs around the edges $D_\gamma(\pm)$:

$$\int_{D_\gamma(\pm)} |\text{grad}\, u|^2 \, dv < \infty, \quad u(M) = u_j(M), \quad M \in D_j \tag{24}$$

This modeling technique is used in [17-20] to calculate the frequency of perfectly thin impermeable screens with ribs. In [12, 21-23] this technique is developed for thin-walled translucent screens (disc, sphere with a hole, ending cylinder) in the case of low-frequency of both electric and magnetic waves. Let consider boundary conditions (18) and (19) as basic boundary conditions for solution of the original problem specified by eqs. (5), (6), (8) and (11).

4. Transformation of boundary conditions

Let us represent unknown functions v_j' in the integral form:

$$v_1' = \int_0^\infty x(\lambda) \frac{I_1(\lambda\rho)}{I_1(\lambda R)} \cos(\lambda z)\, d\lambda, \quad 0 \le \rho < R \tag{25}$$

$$v_2' = \int_0^\infty y(\lambda) \frac{K_1(\lambda\rho)}{K_1(\lambda R)} \cos(\lambda z)\, d\lambda, \quad \rho > R \tag{26}$$

where $x(\lambda)$, $y(\lambda)$ are unknown functions, $I_m(\lambda R)$, $K_m(\lambda R)$ are modified Bessel functions [24].

Substituting integrals (26) into (20), we obtain the integral representation:

$$f_j(z) = \int_0^\infty \big(x(\lambda)g_j(\lambda) + y(\lambda)q_j(\lambda)\big)\cos(\lambda z)\, d\lambda, \quad j = 1,2, \quad |z| < \ell \tag{27}$$

where

$$g_1(\lambda) = \lambda R^2 \left(Q_1 I_1'(\lambda R_1) + \lambda I_1(\lambda R_1) \right) \big/ I_1(\lambda R) \tag{28}$$

$$q_1(\lambda) = \lambda R^2 \left(Q_2 K_1'(\lambda R_2) - \lambda K_1(\lambda R_2) \right) \big/ K_1(\lambda R) \tag{29}$$

$$g_2(\lambda) = P_1\left(\lambda^2 + \frac{1}{R_1^2}\right)\frac{I_1(\lambda R_1)}{I_1(\lambda R)}, \quad q_2(\lambda) = P_2\left(\lambda^2 + \frac{1}{R_2^2}\right)\frac{K_1(\lambda R_2)}{K_1(\lambda R)} \tag{30}$$

$$K_1'(x) = -0{,}5\big(K_0(x) + K_2(x)\big), \quad I_1'(x) = 0{,}5\big(I_0(x) + I_2(x)\big) \tag{31}$$

Let us use the integral transformation:

$$f(z) = \int_0^\infty \tilde{f}(\lambda)\cos\lambda z\, d\lambda, \quad \tilde{f}(\lambda) = \frac{2}{\pi}\int_0^\infty f(z)\cos\lambda z\, dz \tag{32}$$

Let's approximate functions (20) and (21) on interval $0 \le z < \infty$:

$$F_j(z) = \begin{cases} f_j(z), & 0 \le z < \ell \\ 0, & \ell < z < \infty \end{cases} = \int_0^\infty \tilde{F}_j(\lambda)\cos\lambda z\, d\lambda \tag{33}$$

Applying the inverse integral transformation of eq. (32) to eq. (33) and considering (27), it gives:

$$
\begin{aligned}
\tilde{F}_j(\lambda) &= \frac{2}{\pi} \int_0^{\ell} f_j(z) \cos \lambda z \, dz = \\
&= \frac{2}{\pi} \int_0^{\infty} \big(x(\eta) g_j(\eta) + y(\eta) q_j(\eta) \big) \int_0^{\ell} \cos \eta z \cos \lambda z \, dz \, d\eta = \\
&= \int_0^{\infty} \big(x(\eta) g_j(\eta) + y(\eta) q_j(\eta) \big) L(\lambda, \eta) \, d\eta
\end{aligned}
\tag{34}
$$

where

$$
L(\lambda, \eta) = \left(\frac{\sin(\eta - \lambda)\ell}{\eta - \lambda} + \frac{\sin(\eta + \lambda)\ell}{\eta + \lambda} \right) / \pi
\tag{35}
$$

Let $c_j(z)$ be piecewise constant function:

$$
c_j(z) = \begin{cases} c_j, & 0 \leq z < \ell \\ 0, & \ell < z < \infty \end{cases} = c_j \int_0^{\infty} q(\lambda) \cos \lambda z \, d\lambda
\tag{36}
$$

where $q(\lambda) = 2 \sin(\lambda \ell) / (\pi \lambda)$.

Using functions (33) and (36), we combine boundary conditions (18) and (19) as:

$$
\left(v_2' - v_1' \right) \big|_{\rho=R} = F_1(z) + c_1(z), \quad 0 \leq z < \infty
\tag{37}
$$

$$
\left(\frac{\partial v_2'}{\partial \rho} - \frac{\partial v_1'}{\partial \rho} \right) \bigg|_{\rho=R} = F_2(z) + c_2(z), \quad 0 \leq z < \infty
\tag{38}
$$

Let us transform the solution of the problem (eqs. (3), (6), (8), (11)) to the solution of integral Fredholm equations of the second kind, with boundary conditions (38). Functions (26), (32) and (34) occurring in eq. (38) are written in the form of Fourier integrals (32). Applying the inverse integral transform to the eq. (38), we obtain:

$$
y(\lambda) - x(\lambda) = \tilde{F}_1(\lambda) + c_1 q(\lambda)
\tag{39}
$$

$$\lambda \left(y(\lambda) \frac{K_1'(\lambda R)}{K_1(\lambda R)} - x(\lambda) \frac{I_1'(\lambda R)}{I_1(\lambda R)} \right) = \tilde{F}_2(\lambda) + c_2 q(\lambda) \tag{40}$$

Let us solve set of eqs. (39) and (40) in variables $x(\lambda)$, $y(\lambda)$. It gives:

$$\left. \begin{aligned} x(\lambda) &= R \left(\lambda \frac{K_1'(\lambda R)}{K_1(\lambda R)} \tilde{F}_1(\lambda) - \tilde{F}_2(\lambda) \right) + R q(\lambda) \left(\lambda \frac{K_1'(\lambda R)}{K_1(\lambda R)} C_1 - C_2 \right) \\ y(\lambda) &= R \left(\lambda \frac{I_1'(\lambda R)}{I_1(\lambda R)} \tilde{F}_1(\lambda) - \tilde{F}_2(\lambda) \right) + R q(\lambda) \left(\lambda \frac{I_1'(\lambda R)}{I_1'(\lambda R)} C_1 - C_2 \right) \end{aligned} \right\} \tag{41}$$

Substituting integrals (34) in (41), we obtain set of integral Fredholm equations of the second kind:

$$\left. \begin{aligned} x(\lambda) &= R \int_0^\infty \left[x(\eta) \left(\lambda \frac{K_1'(\lambda R)}{K_1(\lambda R)} g_1(\eta) - g_2(\eta) \right) + y(\eta) \left(\lambda \frac{K_1'(\lambda R)}{K_1(\lambda R)} q_1(\eta) - q_2(\eta) \right) \right] \times \\ &\quad \times L(\lambda, \eta) d\eta + R q(\lambda) \left(\lambda \frac{K_1'(\lambda R)}{K_1(\lambda R)} C_1 - C_2 \right) \\ y(\lambda) &= R \int_0^\infty \left[x(\eta) \left(\lambda \frac{I_1'(\lambda R)}{I_1(\lambda R)} g_1(\eta) - g_2(\eta) \right) + y(\eta) \left(\lambda \frac{I_1'(\lambda R)}{I_1(\lambda R)} q_1(\eta) - q_2(\eta) \right) \right] \times \\ &\quad \times L(\lambda, \eta) d\eta + R q(\lambda) \left(\lambda \frac{I_1'(\lambda R)}{I_1(\lambda R)} C_1 - C_2 \right) \end{aligned} \right\} \tag{42}$$

Let us multiply eq. (42) by a factor $\exp(-\lambda h / 4)$. We introduce dimensionless variables of integration $\bar{\lambda} = \lambda R$, $\bar{\eta} = \eta R$ and unknown functions:

$$\begin{aligned} \bar{x}(\bar{\lambda}) &= x(\bar{\lambda} / R) \exp(-\bar{\lambda} h / 4R) / H_0 R^2 \\ \bar{y}(\bar{\lambda}) &= y(\bar{\lambda} / R) \exp(-\bar{\lambda} h / 4R) / H_0 R^2 \end{aligned} \tag{43}$$

The set of dimensionless equations is defined by:

$$\left.\begin{array}{l} \overline{x}(\overline{\lambda}) + \int_0^\infty \left(\overline{K}_{11}(\overline{\lambda},\overline{\eta})\,\overline{x}(\overline{\eta}) + \overline{K}_{12}(\overline{\lambda},\overline{\eta})\,\overline{y}(\overline{\eta}) \right) d\overline{\eta} = \overline{f}(\overline{\lambda}), \;\; 0 \le \overline{\lambda} < \infty \\[2ex] \overline{y}(\overline{\lambda}) + \int_0^\infty \left(\overline{K}_{21}(\overline{\lambda},\overline{\eta})\,\overline{x}(\overline{\eta}) + \overline{K}_{22}(\overline{\lambda},\overline{\eta})\,\overline{y}(\overline{\eta}) \right) d\overline{\eta} = \overline{g}(\overline{\lambda}), \;\; 0 \le \overline{\lambda} < \infty \end{array}\right\} \tag{44}$$

where

$$\overline{K}_{11}(\overline{\lambda},\overline{\eta}) = \left(\overline{g}_2(\overline{\eta}) - \overline{\lambda}\,\overline{g}_1(\overline{\eta}) \frac{K_1'(\overline{\lambda})}{K_1(\overline{\lambda})} \right) \exp\left((\overline{\eta} - \overline{\lambda})\overline{h}/4 \right) \overline{L}(\overline{\lambda},\overline{\eta}) \tag{45}$$

$$\overline{K}_{12}(\overline{\lambda},\overline{\eta}) = \left(\overline{q}_2(\overline{\eta}) - \overline{\lambda}\,\overline{q}_1(\overline{\eta}) \frac{K_1'(\overline{\lambda})}{K_1(\overline{\lambda})} \right) \exp\left((\overline{\eta} - \overline{\lambda})\overline{h}/4 \right) \overline{L}(\overline{\lambda},\overline{\eta}) \tag{46}$$

$$\overline{K}_{21}(\overline{\lambda},\overline{\eta}) = \left(\overline{g}_2(\overline{\eta}) - \overline{\lambda}\,\overline{g}_1(\overline{\eta}) \frac{I_1'(\overline{\lambda})}{I_1(\overline{\lambda})} \right) \exp\left((\overline{\eta} - \overline{\lambda})\overline{h}/4 \right) \overline{L}(\overline{\lambda},\overline{\eta}) \tag{47}$$

$$\overline{K}_{22}(\overline{\lambda},\overline{\eta}) = \left(\overline{q}_2(\overline{\eta}) - \overline{\lambda}\,\overline{q}_1(\overline{\eta}) \frac{I_1'(\overline{\lambda})}{I_1(\overline{\lambda})} \right) \exp\left((\overline{\eta} - \overline{\lambda})\overline{h}/4 \right) \overline{L}(\overline{\lambda},\overline{\eta}) \tag{48}$$

$$\overline{f}(\overline{\lambda}) = \frac{2}{\pi} \sin(\overline{\lambda}\,\ell) \left(\frac{K_1'(\overline{\lambda})}{K_1(\overline{\lambda})} \overline{C}_1 - \frac{\overline{C}_2}{\overline{\lambda}} \right) \exp(-\overline{\lambda}\,\overline{h}/4) \tag{49}$$

$$\overline{g}(\overline{\lambda}) = \frac{2}{\pi} \sin(\overline{\lambda}\,\ell) \left(\frac{I_1'(\overline{\lambda})}{I_1(\overline{\lambda})} \overline{C}_1 - \frac{\overline{C}_2}{\overline{\lambda}} \right) \exp(-\overline{\lambda}\,\overline{h}/4) \tag{50}$$

$$\overline{g}_1(\overline{\lambda}) = \overline{\lambda} \left(\overline{Q}_1 I_1'(\overline{\lambda}\overline{R}_1) + \overline{\lambda} I_1(\overline{\lambda}\overline{R}_1) \right) \big/ I_1(\overline{\lambda}) \tag{51}$$

$$\overline{q}_1(\overline{\lambda}) = \overline{\lambda} \left(\overline{Q}_2 K_1'(\overline{\lambda}\overline{R}_2) - \overline{\lambda} K_1(\overline{\lambda}\overline{R}_2) \right) \big/ K_1(\overline{\lambda}) \tag{52}$$

$$\overline{g}_2(\overline{\lambda}) = \left(1 + \overline{\lambda}^2 \overline{R}_1^2 \right) \frac{\overline{P}_1 I_1(\overline{\lambda}\overline{R}_1)}{\overline{R}_1^2 I_1(\overline{\lambda})} \tag{53}$$

$$\overline{q}_2\left(\overline{\lambda}\right) = \left(1 + \overline{\lambda}^2 \overline{R}_2^2\right) \frac{\overline{P}_2 K_1\left(\overline{\lambda}\overline{R}_2\right)}{\overline{R}_2^2 K_1\left(\overline{\lambda}\right)} \tag{54}$$

$$\overline{L}\left(\overline{\lambda},\overline{\eta}\right) = \left(\frac{\sin\left(\overline{\eta} - \overline{\lambda}\right)\overline{\ell}}{\overline{\eta} - \overline{\lambda}} + \frac{\sin\left(\overline{\eta} + \overline{\lambda}\right)\overline{\ell}}{\overline{\eta} + \overline{\lambda}}\right) / \pi \tag{55}$$

where

$$\overline{h} = h / R, \quad \overline{\Delta} = \Delta / R, \quad \overline{R}_j = R_j / R, \quad \overline{\ell} = \ell / R, \quad \overline{Q}_j = Q_j R, \quad \overline{P}_j = P_j / R,$$

$$\overline{C}_1 = \overline{Q}_1 + \overline{Q}_2 + \overline{\Delta} / \overline{R}_1 \overline{R}_2, \quad \overline{C}_2 = \overline{P}_1 / \overline{R}_1 + \overline{P}_2 / \overline{R}_2.$$

5. Calculation of a screening coefficient

Let us calculate the magnetic field at the center of shell D at the point O. We obtain

$$\left.\vec{H}_1\right|_{\substack{z=0 \\ \rho=0}} = -\operatorname{grad} u_1\Big|_{\substack{z=0 \\ \rho=0}} = -H_0\vec{e}_x - \frac{1}{2}\int_0^\infty \frac{\lambda x(\lambda)}{I_1(\lambda R)}d\lambda\,\vec{e}_x = -H_0\left(1 + L\left(\overline{x}\left(\overline{\lambda}\right)\right)\right)\vec{e}_x \tag{56}$$

where

$$L\left(\overline{x}\left(\overline{\lambda}\right)\right) = \frac{1}{2}\int_0^\infty \frac{\overline{\lambda}\,\overline{x}\left(\overline{\lambda}\right)}{I_1\left(\overline{\lambda}\right)}\exp\left(\overline{\lambda}\overline{h} / 4\right)d\overline{\lambda} \tag{57}$$

The screening coefficient at the center of cylindrical shell is

$$K_e = \left|\vec{H}_1\right| / \left|\vec{H}_0\right|_{\substack{z=0 \\ \rho=0}} = \left|1 + L\left(\overline{x}\left(\overline{\lambda}\right)\right)\right| \tag{58}$$

The efficiency coefficient of the shell is defined as

$$E = 1 / K_e \tag{59}$$

Considering the integral on finite the interval of integration from O to A and applying the Simpson generalized quadrature formula of fourth-order accuracy to the set of integral Fredholm eq. (44) let's transform the set of integral Fredholm equations into a set of linear algebraic equations [22, 25]:

$$x0_n + \sum_{k=0}^{N} A_k (K_{nk}^{11} x0_k + K_{nk}^{12} y0_k) = f1_n$$

$$y0_n + \sum_{k=0}^{N} A_k (K_{nk}^{21} x0_k + K_{nk}^{22} y0_k) = g1_n, \quad n = 0, 1, ..., N$$

$$(60)$$

where $\quad K_{nk}^{11} = \overline{K}_{11}(\overline{\lambda}_n, \overline{\lambda}_k), \qquad K_{nk}^{12} = \overline{K}_{12}(\overline{\lambda}_n, \overline{\lambda}_k), \qquad K_{nk}^{21} = \overline{K}_{21}(\overline{\lambda}_n, \overline{\lambda}_k),$
$K_{nk}^{22} = \overline{K}_{22}(\overline{\lambda}_n, \overline{\lambda}_k), \quad f1_n = \overline{f}(\overline{\lambda}_n), \quad g1_n = \overline{g}(\overline{\lambda}_n), \quad \overline{\lambda}_n = nh, \quad h = A/N,$

where N is an even number, A_k are weighting coefficients calculated by the formulas:

$$A_0 = A_N = h/3, \quad A_{2j} = 2h/3, \quad A_{2j+1} = 4h/3, \quad j = 1, 2, ..., N/2 - 1 \quad (61)$$

The solution of the set of eq. (60) $x0_n$, $y0_n$, $n = 0, 1, ..., N$, are taken as approximate solution of the set of integral eq. (44) in the points $\overline{\lambda}_n$. Calculations showed that to obtain solutions of the set of eq. (60) with accuracy of 10^{-3} for considered parameters of the problem we need $A = 8$, $h = 0,01$. Input parameters for the computational experiment: $n = 2m$ is a number of layers, where $m = 20$ is a number of pairs of layers; $\Delta_1 = \Delta_3 = ... = \Delta_{n-1} = h_{sum}/m$ are thicknesses of magnetic layers, where $h_{sum} = 4,8 \cdot 10^{-4}\, m$ is a total thickness of magnetic layer; $\Delta_2 = \Delta_4 = ... = \Delta_n = \alpha \Delta_1$ are thicknesses of nonmagnetic layers, where $\alpha = 0,125; 0,25; 0,5;$ $\mu_r^{(1)} = \mu_r^{(3)} = ... = \mu_r^{(n-1)} = 10^4$ are relative permeabilities of magnetic layers; $\varepsilon_r^{(1)} = \varepsilon_r^{(3)} = ... = \varepsilon_r^{(n-1)} = i\gamma_1/\omega\varepsilon_0$ are relative permittivities of magnetic layers, where $\gamma_1 = 10^7$ S/m; $\mu_r^{(2)} = \mu_r^{(4)} = ... = \mu_r^{(n)} = 1$ are relative permeabilities of nonmagnetic layers; $\varepsilon_r^{(2)} = \varepsilon_r^{(4)} = ... = \varepsilon_r^{(n)} = i\gamma_2/\omega\varepsilon_0$ are relative permittivities of nonmagnetic layers, where $\gamma_2 = 6 \cdot 10^7$ S/m; $\ell = 0,5$ m is half-length of the cylinder; $R_1 = 1,1 \cdot 10^{-2}$ m is an inner radius of the cylinder; $\Delta_0 = 10^{-3}$ m is thickness of nonmagnetic null layer; $\mu_r^{(0)} = 1$ is relative permeability of nonmagnetic null layer; $\varepsilon_r^{(0)} = i\gamma_2/(\omega\varepsilon_0)$ is relative permittivity of nonmagnetic null layer. Figures 2. and 3. show the relation between the efficiency coefficient E and a number of layers n for some frequencies of magnetic field.
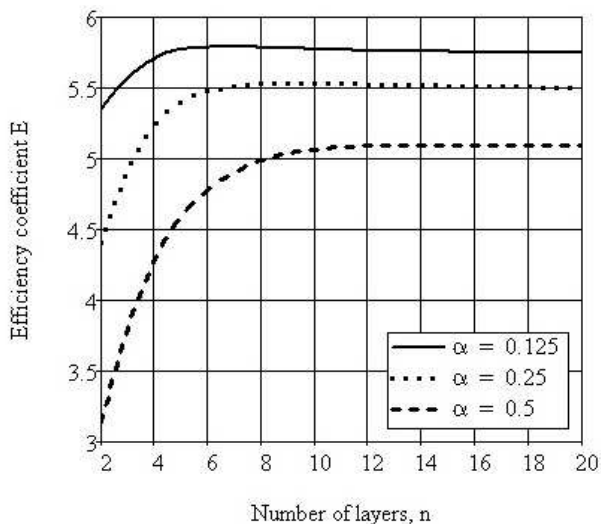
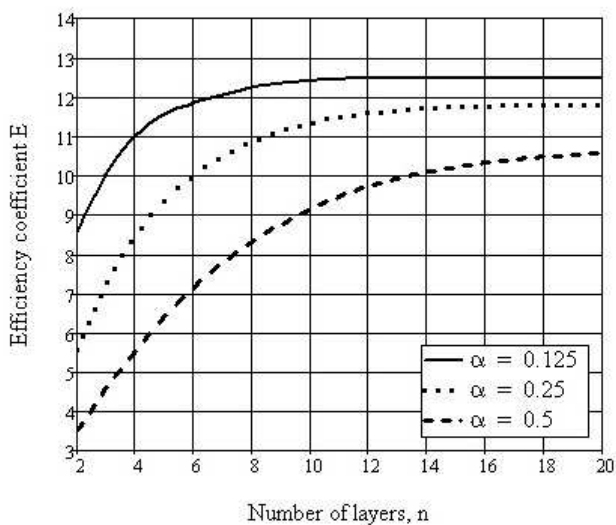Fig. 2. The relation between the efficiency coefficient E and a number of layers n, $f = 200$ Hz

Fig. 3. The relation between the efficiency coefficient E and a number of layers n, $f = 500$ Hz

6. Conclusions

The mathematical model of non-local bilateral boundary conditions of multilayer finite cylindrical shells was built. These non-local bilateral boundary

conditions describe the propagation of a low-frequency electromagnetic field across layers of the shell. The method of transformation of the boundary conditions to the set of integral Fredholm equations of the second kind was presented. Analytical transformations are based on the Bessel functions. The value of the efficiency coefficient of the shell was also calculated. The results showed that the efficiency coefficient increases with a number of layers for all values of frequencies of magnetic field. The results of mathematic simulation can be used in multilayer shell manufacturing technology.

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PRZENIKALNOŚĆ FAL ELEKTROMAGNETYCZNYCH O NISKIEJ CZĘSTOTLIWOŚCI PRZEZ WIELOWARSTWOWĄ POWŁOKĘ WALCOWĄ

S t r e s z c z e n i e

W artykule przedstawiono wyniki analizy przenikania fal elektromagnetycznych o niskiej częstotliwości przez wielowarstwową powłokę walcową. Zbudowano model matematyczny nielokalnych dwustronnych warunków brzegowych wielowarstwowych powłok walcowych. Te dwustronne nielokalne warunki brzegowe opisują przenikanie pola elektromagnetycznego o niskiej częstotliwości przez warstwy powłoki. Opracowano sposób przekształcania równań brzegowych do całkowych równań Fredholma drugiego rodzaju. Ponadto obliczono wartość współczynnika sprawności powłoki. Do przekształceń analitycznych wykorzystano funkcje Bessela.

Słowa kluczowe: funkcja Bessela, warunki brzegowe, fale elektromagnetyczne, pole elektromagnetyczne