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Inequality for Polynomials with Prescribed Zeros

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ABSTRACT: For a polynomial $p(z)$ of degree n with a zero at β, of order at least $k(\geq 1)$, it is known that

$$\int_0^{2\pi}\left|\frac{p(e^{i\theta})}{(e^{i\theta}-\beta)^k}\right|^2 d\theta \leq \left\{\prod_{j=1}^{k}\left(1+|\beta|^2-2|\beta|\cos\frac{\pi}{n+2-j}\right)\right\}^{-1}\int_0^{2\pi}|p(e^{i\theta})|^2 d\theta.$$

By considering polynomial $p(z)$ of degree n in the form

$p(z) = (z-\beta_1)(z-\beta_2)\ldots(z-\beta_k)q(z)$, $k \geq 1$ and $q(z)$, a polynomial of degree $n-k$, with

$$S = \{\ \gamma_{l_1}\gamma_{l_2}\ldots\gamma_{l_k} : \gamma_{l_1}\gamma_{l_2}\ldots\gamma_{l_k} \text{ is a permutation of } k \text{ objects } \beta_1,\beta_2,\ldots,\beta_k \text{ taken all at a time}\},$$

we have obtained

$$\int_0^{2\pi}\left|\frac{p(e^{i\theta})}{(e^{i\theta}-\beta_1)(e^{i\theta}-\beta_2)\ldots(e^{i\theta}-\beta_k)}\right|^2 d\theta$$
$$\leq \left[\min_{\gamma_{l_1}\gamma_{l_2}\cdots\gamma_{l_k}\in S}\left\{\prod_{j=1}^{k}\left(1+|\gamma_{l_j}|^2-2|\gamma_{l_j}|\cos\frac{\pi}{n+2-j}\right)\right\}^{-1}\right]\int_0^{2\pi}|p(e^{i\theta})|^2 d\theta,$$

a generalization of the known result.

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