The Real and Complex Convexity

Abidi Jamel

ABSTRACT: We prove that the holomorphic differential equation $\varphi''(\varphi+c) = \gamma(\varphi')^2$ ($\varphi : \mathbb{C} \to \mathbb{C}$ be a holomorphic function and $(\gamma, c) \in \mathbb{C}^2$) plays a classical role on many problems of real and complex convexity. The condition exactly $\gamma \in \{1, \frac{s-1}{s} / s \in \mathbb{N}\backslash\{0\}\}$ (independently of the constant c) is of great importance in this paper.

On the other hand, let $n \geq 1$, $(A_1, A_2) \in \mathbb{C}^2$, and $g_1, g_2 : \mathbb{C}^n \to \mathbb{C}$ be two analytic functions. Put $u(z,w) =\mid A_1 w - g_1(z) \mid^2 + \mid A_2 w - g_2(z) \mid^2$, $v(z,w) =\mid A_1 w - \overline{g_1}(z) \mid^2 + \mid A_2 w - \overline{g_2}(z) \mid^2$, for $(z,w) \in \mathbb{C}^n \times \mathbb{C}$. We prove that u is strictly plurisubharmonic and convex on $\mathbb{C}^n \times \mathbb{C}$ if and only if $n = 1$, $(A_1, A_2) \in \mathbb{C}^2\backslash\{0\}$ and the functions g_1 and g_2 have a classical representation form described in the present paper.

Now v is convex and strictly psh on $\mathbb{C}^n \times \mathbb{C}$ if and only if $(A_1, A_2) \in \mathbb{C}^2\backslash\{0\}$, $n \in \{1, 2\}$ and g_1, g_2 have several representations investigated in this paper.