On the Derivative of a Polynomial with Prescribed Zeros

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ABSTRACT: For a polynomial $p(z) = a_n \prod_{t=1}^{n}(z - z_t)$ of degree n having all its zeros in $|z| \leq K$, $K \geq 1$ it is known that

$$\max_{|z|=1} |p'(z)| \geq \frac{2}{1+K^n} \left\{ \sum_{t=1}^{n} \frac{K}{K+|z_t|} \right\} \max_{|z|=1} |p(z)| \ .$$

By assuming a possible zero of order m, $0 \leq m \leq n-4$, at $z = 0$, of $p(z)$ for $n \geq k+m+1$ with integer $k \geq 3$ we have obtained a new refinement of the known result.