

ABSTRACT: In this paper we apply Rothe's Fixed Point Theorem to prove the interior approximate controllability of the following semilinear impulsive Heat Equation

$$\begin{cases} z_t = \Delta z + 1_\omega u(t, x) + f(t, z, u(t, x)), & \text{in } (0, \tau] \times \Omega, t \neq t_k \\ z = 0, & \text{on } (0, \tau) \times \partial\Omega, \\ z(0, x) = z_0(x), & x \in \Omega, \\ z(t_k^+, x) = z(t_k^-, x) + I_k(t_k, z(t_k, x), u(t_k, x)), & x \in \Omega, \end{cases}$$

where $k = 1, 2, \dots, p$, Ω is a bounded domain in \mathbb{R}^N ($N \geq 1$), $z_0 \in L_2(\Omega)$, ω is an open nonempty subset of Ω , 1_ω denotes the characteristic function of the set ω , the distributed control u belongs to $C([0, \tau]; L_2(\Omega))$ and $f, I_k \in C([0, \tau] \times \mathbb{R} \times \mathbb{R}; \mathbb{R})$, $k = 1, 2, 3, \dots, p$, such that

$$|f(t, z, u)| \leq a_0 |z|^{\alpha_0} + b_0 |u|^{\beta_0} + c_0, \quad u \in \mathbb{R}, z \in \mathbb{R}.$$

$$|I_k(t, z, u)| \leq a_k |z|^{\alpha_k} + b_k |u|^{\beta_k} + c_k, \quad k = 1, 2, 3, \dots, p, \quad u \in \mathbb{R}, z \in \mathbb{R}.$$

with $\frac{1}{2} \leq \alpha_k < 1$, $\frac{1}{2} \leq \beta_k < 1$, $k = 0, 1, 2, 3, \dots, p$. Under this condition we prove the following statement: For all open nonempty subsets ω of Ω the system is approximately controllable on $[0, \tau]$. Moreover, we could exhibit a sequence of controls steering the nonlinear system from an initial state z_0 to an ϵ neighborhood of the final state z_1 at time $\tau > 0$.