

ABSTRACT: Kuniyeda, Montel and Toya had shown that the polynomial $p(z) = \sum_{k=0}^n a_k z^k; a_0 \neq 0$, of degree n , does not vanish in

$$|z| \leq \{1 + (\sum_{j=1}^n |a_j/a_0|^p)^{q/p}\}^{-1/q},$$

where $p > 1, q > 1, (1/p) + (1/q) = 1$ and we had proved that $p(z)$ does not vanish in $|z| \leq \alpha^{1/q}$, where

$$\begin{aligned} \alpha &= \text{unique root in } (0, 1) \text{ of } D_n x^3 - D_n S x^2 + (1 + D_n S)x - 1 = 0, \\ D_n &= (\sum_{j=1}^n |a_j/a_0|^p)^{q/p}, \\ S &= (|a_1| + |a_2|)^q (|a_1|^p + |a_2|^p)^{-(q-1)}, \end{aligned}$$

a refinement of Kuniyeda et al.'s result under the assumption

$$D_n < (2 - S)/(S - 1).$$

Now we have obtained a generalization of our old result and proved that the function

$$f(z) = \sum_{k=0}^{\infty} a_k z^k, (\neq \text{a constant}); a_0 \neq 0,$$

analytic in $|z| \leq 1$, does not vanish in $|z| < \alpha_m^{1/q}$, where

$$\begin{aligned} \alpha_m &= \text{unique root in } (0, 1) \text{ of } D x^{m+1} - D M_m x^2 + (1 + D M_m)x - 1 = 0, \\ D &= (\sum_{k=1}^{\infty} |a_k/a_0|^p)^{q/p}, \\ M_m &= (\sum_{k=1}^m |a_k|)^q (\sum_{k=1}^m |a_k|^p)^{-q/p}, \end{aligned}$$

m = any positive integer with the characteristic that there exists a positive integer $k(\leq m)$ with $a_k \neq 0$.