IMPLICATIONS OF DISCOUNTING METHODS AND RELATIONS BETWEEN NPV, IRR AND MIRR FOR EFFICIENCY EVALUATION OF INVESTMENT PROJECTS

Efficiency evaluation of investment projects in market economy ought to be based mainly on the use of indices based on discount method. These indices, in spite of being known for many years, in many cases are used incorrectly. The discount technique used in the indices requires the selection of a uniform moment of time for which cash flows are discounted as well as the inclusion of the total value of planned/realized investment along with its residual value. The use of the aforementioned indices without proper knowledge regarding not only economics but mathematics as well can result in an inappropriate efficiency evaluation of analyzed investment projects and, as a consequence, lead to undertaking wrong decisions that may put the company at the risk of making considerable losses. This article outlines the consequences of choosing discounting for various moments of time for discounted indices of efficiency evaluation of investments – NPV, IRR and MIRR. It also presents mathematical relations between these methods and the consequences of such relations for the evaluation of investment profitability. The article concludes that as for IRR method, it is of no significance for what period we discount as IRR values, irrespectively of the selected moment of time, will be always equal. Nonetheless, this does not apply to the use of a modified version of IRR method, i.e. MIRR. Here the choice of the moment for which cash surplus will be discounted is of no significance in only one case, i.e. when MIRR equals IRR. The applied conception of calculating MIRR ought to result from the accepted conception of calculating NPV; nonetheless, it does not function this way in practice.

Keywords: investments, investment efficiency, investment project evaluation methods

1. INTRODUCTION

Discounting methods include the time variability of money value. These methods use the method of discount which brings cash surplus value from various years to the present value in the base year so it is possible to compare them in time. The most frequently used in economy practice investment efficiency measures that use discount account are NPV methods (net present value) and IRR (internal rate of return) as well as its modified version MIRR (modified internal rate of return) [8], [10], [18], [19], [23], [24], [27], [28]. These methods, though very simple as far as their mathematical construction is concerned, due to assumed theses may vary to such a substantial degree that an analysis conducted with their use may lead to erroneous conclusions.
2. RESEARCH RESULTS AND DISCUSSION

NPV is defined as the sum of discounted for a particular moment difference between revenues and expenditure connected with an investment project. In literature one may find various formula forms of this index:

a) the sum of discounted cash flow, discounted from t=0 [9], [11], [21], [25], [26], [29]:

$$NPV = \sum_{t=0}^{n} \frac{NCF_t}{(1+r)^t}$$  \hspace{1cm} (1)

b) the difference between the sum of discounted cash flow and discounted investment outlays, discounted from period t=0 [16], [21], [29]:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} - \sum_{t=0}^{m} \frac{I_t}{(1+r)^t}$$  \hspace{1cm} (2)

which de facto is an alternative to formula (1), or when the total sum of outlays is incurred in one year – year zero (t0) as:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} - I_0$$  \hspace{1cm} (3)

c) the sum of discounted net cash flow from period t=1 [7], [20]:

$$NPV = \sum_{t=1}^{n} \frac{NCF_t}{(1+r)^t}$$  \hspace{1cm} (4)

d) the difference between discounted cash flow from period t=1 and not discounted investment outlays [15], [17]:

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - I$$  \hspace{1cm} (5)

e) the difference between discounted net cash flow and investment outlays from period t=0 [1], [32]:

$$NPV = \sum_{t=1}^{n} \frac{NCF_t}{(1+r)^t} - \sum_{t=1}^{m} \frac{I_t}{(1+r)^t}$$  \hspace{1cm} (6)

f) the difference between discounted cash flow and investment outlays from period t=1 [22], [35], [36], [37], [38]:

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - \sum_{t=1}^{m} \frac{I_t}{(1+r)^t}$$  \hspace{1cm} (7)

while if the total sum of outlays is incurred in one year – year zero (t0), as:
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\[ NPV = \sum_{i=0}^{n} \frac{CF_i}{(1+r)^i} - I_0 \]  

(8)

Some of these formulas are very similar to others and an unskilled researcher may not notice the differences between them. Nevertheless, they are highly significant and the failure to notice them can result in an incorrectly conducted efficiency analysis with the use of this index.

Formula (1) and (2) are de facto, according to the cited authors, two alternatives. Formula (1) can be written as follows:

\[ NPV = NCF_0 a_0 + NCF_1 a_1 + NCF_2 a_2 + ... + NCF_n a_n \]  

(9)

where:
a – discount factor;
whereas formula (2):

\[ NPV = CF_0 a_0 + CF_1 a_1 + ... + CF_n a_n - (I_0 a_0 + I_1 a_1 + ... + I_m a_m) \]  

(10)

\[ NPV = CF_0 a_0 - I_0 a_0 + CF_1 a_1 - I_1 a_1 + ... + CF_n a_n - I_m a_m \]  

(11)

Because NCF=CF-I, formula (10) is equivalent to formula (11). They allow to calculate NPV value when the investment outlay is incurred on a one-off basis as well as when it is not so (formula (3) is nothing else than a special case of formula (1) and (2). However, the moment of discounting cash flow may seem strange; i.e. the end of year zero, or, in other words, the beginning of year one.

One may observe that the first NCF value (NCF_0) is not updated at all (or in other words, updated at the end of year zero) while the other values are updated at the beginning of year one – i.e. the end of year zero.

If outlays are incurred on a one-off basis\(^2\), this moment is the beginning of investment operation, which fully justifies the selection of a particular moment of time. The selection of investment operation beginning facilitates exact estimation of the scale of essential investment outlays (Fig. 1).

\(^2\) Often in literature one uses an alternative statement „one-off incurred investment outlays – construction period shorter than one year”. Such a statement is correct only when the total sum of investment outlays is incurred from one’s own funds. In the case of external financing, for instance a credit, investment outlays are spread out over more than one year, although construction period is under one year.
Nonetheless, if (as assumed in the formula) this formula serves to calculate NPV value, when outlays are not incurred on a one-off basis, this moment is not a special one in the period of investment functioning (Fig. 2).

However, having in mind the fact of bringing all values to one moment of time, they are of course summable, so the formulas are mathematically correct.

Formula (4) and its equivalent formula (7) are based on discounting cash flow at the beginning of construction period (Fig. 3). It can be developed as follows:

$$NPV = NCF_1a_1 + NCF_2a_2 + \ldots + NCF_na_n$$

(12)
As compared with other methods, its advantage is the fact that irrespectively of the period of outlays (one-off or longer), this period on which cash flow is updated can be always identified in one way – as the beginning of construction period. It was not so in previous cases - when investment outlays were incurred on a one-off basis – it was the beginning of operation period, whereas when the outlays were spread out over a longer period of time, this moment could not be logically estimated – it is the beginning of the first year and it is still puzzling why this particular moment was chosen by the authors.

In formula (5) „I” is termed as investment outlays, yet there is no information regarding the period of incurring these costs. This formula is correct only when outlays are incurred on a one-off basis. Then it becomes equivalent to formula (8) and means discounting all cash flows at the beginning of construction period (Fig 4).

Formula (6) is barely acceptable as correct if cash flow net (NCF), understood as the difference between cash flow (CF) and investment outlays I. Taking that investment outlays in this formula are clearly distinguished in the form of \( \sum_{i=1}^{m} \frac{I_i}{(1+r)^i} \), the first fraction of the formula can include only CF. Such a notation would cause double substraction of investment outlays, which would give an erroneous NPV value.

As mentioned before, taking into consideration the possibility to distinguish preliminary investment outlays in discount formula, it is optimal to discount all elements of NCF at the beginning of operation period. Such a solution is proposed by K. Leszczyński [14]. He argues the simultaneous use of discount method and capitalization in NPV technique. The result is capitalization of investment outlays incurred in particular years and discounting CF, which leads to an update of all components at the beginning of operation period – the end of construction period (Fig. 5).

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4 One may assume that the intention of the authors was the update at the beginning of operation period, as it is in the case of formula (3), and formula (1) and (2) were created as a generalized form of formula (3) (there, however, this condition is not fulfilled). Nonetheless, such attitude does not seem right as the situation when outlays are incurred on a one-off basis is a special case of a situation when it is not so, formula (3) ought to be a special case of formula (1) and (2) and lead to the same conclusions.
Fig. 5. Discounting at the beginning of operation period (outlays incurred over a few years)

Such a conception would be fully justified if the period of incurring investment outlays equaled the construction period. Nonetheless, it is possible that during construction period there might appear investment operation. As a result, there would be discrepancies between the term of the end of construction period and the beginning of operation period, consequently it would be extremely difficult to use this conception.

One ought to stress that formulas to calculate NPV proposed in various sources very rarely include the RV (residual value). Applying formula (7) and including RV, the formula serving to calculate NPV can be written as:

$$NPV = \sum_{t=1}^{m} \frac{CF_t}{(1+r)^t} - \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} + \frac{RV}{(1+r)^t}$$

It is a very significant component of NPV equation and failure to include it may lead to an erroneous efficiency evaluation of a given investment project – the shorter discount period we assume, the greater RV remains. It is certainly important to bear in mind that comparison of investment projects using NPV criterion requires identical discounting periods. Otherwise calculated NPV values will be incomparable. Concluding, one faces a problem with choosing the length of discount period. Often it is assumed that discount period should be equal to the period of project realization and operation. As it is difficult to precisely estimate these values, it is recommended to choose discount period equal to operation period of the component that is of key value to a given business activity [11]. Applying such premises when selecting discount period does not seem correct, though. For various investments such components may be completely different with various operation periods, which may lead to choosing a different discount period for various investments, which is has been acknowledged as a substantially wrong assumption when using NPV index. Therefore, it would be better to choose discount period including the start-up period and a few years of regular operation of a facility – the remaining value will increase the RV. Another argument supporting such a choice is the increasing level of uncertainty regarding future values of CF components and, accordingly, the risk incurred along with passing time (Fig 6).
Too long discount period may lead to erroneous estimations of cash flow value and, as a result, give NPV value which is significantly different from the real one. Accepting a project to be realized using NPV method means selecting projects which satisfy the equation NPV≥0. It implies that selected projects have profitability higher or equal to minimum, estimated by the assumed discount rate. In case of a number of variants satisfying this inequality, one ought to choose a project with the highest NPV value (NPV→max).

The second method under question is the IRR method (Internal Rate of Return). IRR is defined as a discount rate where the present value of cash flow levels up with investment outlays. Accordingly it is a discount rate where NPV=0 [2], [4], [5], [12], [13], [30], [33], [34]. Using formula (13), one can write:

\[
\sum_{t=1}^{n} \frac{CF_t}{(1 + IRR)^t} - \sum_{t=m}^{n} \frac{I_t}{(1 + IRR)^t} + \frac{RV}{(1 + IRR)^t} = 0
\]  

(14)

Accepting a project to be realized using IRR method occurs when the estimated IRR value is higher or equal to discount rate, referred to as the border profit rate below which it is not profitable to invest in a given MAAR project (minimum acceptable rate of return) IRR≥MARR. When estimating the internal rate of return it is irrelevant on what moment cash flow is updated. In the point where NPV=0, IRR estimated with various formulas using various moments of time, bringing the value of cash flow to one moment of time, it will have the same value (Fig 7).
The very conception of IRR method itself raises doubts of many economists. From its construction one may conclude that retrieved financial resources are reinvested according to the same IRR rate. As a result, it gives an inflated value of rate of return from a given business activity. However, when comparing internal rates of return of a few projects, such doubts are rather unjustified. Reinvesting with IRR rate concerns all researched internal rates of return, so the choice of a project with higher IRR indicates a project with higher efficiency.

These doubts have grounds in the case of discrepancies between evaluation criteria. There the conception regarding reinvesting with internal rate of return may give inflated results. If so, it is advisable to use MIRR method (modified internal rate of return). This method assumes that it is possible to estimate the rate with which financial resources are reinvested [5].

There are many formulas available in sources, alike there are numerous NPV formulas. The most frequently observed form is formula (15) (and its mathematical transformations) [5], [5], [37]:

\[
\sum_{t=0}^{n} COF_t \frac{(1 + r)^{n-t}}{(1 + MIRR)^n} = \sum_{t=0}^{n} CIF_t (1 + r)^{n-t} 
\]  

(15)

where:

- COF – cash outflow,
- CIF – cash inflow.

Relatively rarely one observes formula (16) (and its mathematical transformations) [3], [31]:

\[
\sum_{t=1}^{n} COF_t \frac{(1 + r)^{n-t}}{(1 + MIRR)^n} = \sum_{t=1}^{n} CIF_t (1 + r)^{n-t} 
\]  

(16)

\footnote{Disregarding the differences in symbols used by the authors}
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Used formulations of CIF and COF are not very fortunate – analyzing their mathematical and economic sense as well as the authors’ intentions, CIF do not constitute cash inflow but cash surplus gained in particular years whereas COF constitutes negative values of net cash flow (NCF). More fortunate are terms used by Nowak E. Pielichaty E. and Poszwa M. – NCF+ and NCF− [21]6, i.e. positive and negative net cash flow.

Using these symbols formula (15) can be written as follows:

$$\sum_{t=0}^{n} \frac{NCF_t^-}{(1+r)^t} = \sum_{t=0}^{n} \frac{NCF_t^+(1+r)^{n-t}}{(1+MIRR)^n}$$

and formula (16) as:

$$\sum_{t=1}^{n} \frac{NCF_t^-}{(1+r)^t} = \sum_{t=1}^{n} \frac{NCF_t^+(1+r)^{n-t}}{(1+MIRR)^n}$$

Both formulas in this form allow discounting of cash outflows and capitalization of cash inflow with the rate which is freely estimated by the governing body. However, both formulas differ with regard to moments for which cash outflows are discounted. In formula (14) they are discounted at the end of year zero, whereas in formula (15) at the beginning of construction. Applying these formulas gives different results so it is of importance which formula one applies. These formulas give the same result in only one case – when the rate equals the internal rate IRR, i.e. MIRR will equal IRR (Fig. 8).

Fig. 8. Comparison of MIRR conception depending on the period for which cash outflow is discounted

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6 Unfortunately, MIRR formula in this publication is not mathematically correct
With \( r \) rate lower than IRR of the project, application of formula (15) gives higher results than formula (16). The used conception of MIRR calculation ought to result from the accepted conception of calculating NPV. Nonetheless, it is not often seen in practice.\(^7\) It is strange as MIRR formula (alike IRR) mathematically results from NPV formula. Both formulas do not include the residual value (RV). Including RV, one may write:

\[
\sum_{t=1}^{m} \frac{CF_t}{(1+r)^t} - \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} + \frac{RV}{(1+r)^t} = 0
\]  \hspace{1cm} (19)

\[
\sum_{t=1}^{m} \frac{CF_t}{(1+r)^t} = \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} - \frac{RV}{(1+r)^t}
\]  \hspace{1cm} (20)

multiplying both forms of equation by \((1+r)^n\) one obtains:

\[
\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} (1+r)^n = \left( \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} - \frac{RV}{(1+r)^t} \right) (1+r)^n
\]  \hspace{1cm} (21)

Formula (21) is equivalent to formula (22):

\[
\sum_{t=1}^{n} CF_t (1+r)^{n-t} = \left( \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} - \frac{RV}{(1+r)^t} \right) (1+r)^n
\]  \hspace{1cm} (22)

This way one obtains an equivalent between capitalized cash flows \( CF \) and a product of the term \((1+r)^n\) and discounted at the beginning of construction sum of investment outlays reduced by discounted residual value \( RV \). Rate \( r \) in the term \((1+r)^n\) is the wanted MIRR rate. One may therefore write:

\[
\sum_{t=1}^{n} CF_t (1+r)^{n-t} = \left( \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} - \frac{RV}{(1+r)^t} \right) (1+MIRR)^n
\]  \hspace{1cm} (23)

Dividing both sides by the term \((1+MIRR)^n\) one obtains:

\[
\frac{\sum_{t=1}^{n} CF_t (1+r)^{n-t}}{(1+MIRR)^n} = \sum_{t=1}^{m} \frac{I_t}{(1+r)^t} - \frac{RV}{(1+r)^t}
\]  \hspace{1cm} (24)

which is equivalent to formula (25):

\(^7\) Inconsistencies of discounting for one moment of time are observed not only in literature, but also in Excel Spreadsheet where in formula NPV cash flows are discounted at the beginning of construction period and in MIRR formula at the end of year zero.
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\[ \sum_{t=1}^{n} \frac{I_t}{(1 + r)^t} - \frac{RV}{(1 + r)^t} = \frac{\sum_{t=1}^{n} CF_t (1 + r)^{n-t}}{(1 + MIRR)^n} \]  (25)

The rate used for discounting may, but does not need to, equal the rate used for capitalization. One can therefore write that:

\[ \sum_{t=1}^{n} \frac{I_t}{(1 + r_1)^t} - \frac{RV}{(1 + r_1)^t} = \frac{\sum_{t=1}^{n} CF_t (1 + r_2)^{n-t}}{(1 + MIRR)^n} \]  (26)

where:
- \( r_1 \) – discount rate
- \( r_2 \) – capitalization rate (reinvestment).

In this formula, as compared with formula (17) and (18), the residual value \( RV \) was included. Moreover it needs to be stressed that in the aforementioned formulas preliminary investment outlays \( I \) were replaced with the term of negative net cash flows \( NCF^- \), and cash flows \( CF \) with the term of positive cash flows \( NCF^+ \). Yet, applying such a conception raises some doubts. If debit balance of cash flow occurs, the balance is treated as investment outlay and it is included on the left side of equation, i.e. discounted. According to formula (26) all – either positive or negative balance of cash flow – should be capitalized with \( r_2 \) rate.

Such a modification has its advantages – it points to actual cash surplus which can be reinvested in the company with \( r_2 \) rate. Therefore, modifying this formula (26) one obtains:

\[ \sum_{t=1}^{n} \frac{NCF_t^-}{(1 + r_1)^t} - \frac{RV}{(1 + r_1)^t} = \frac{\sum_{t=1}^{n} NCF_t^+ \cdot (1 + r_2)^{n-t}}{(1 + MIRR)^n} \]  (27)

Formula (27) assures correct calculation of \( MIRR \) index with varied or equal discounting and reinvestment rates. It also includes discounted residual value of the investment.

3. CONCLUSIONS

The majority of formulas provided in literature serving to calculate NPV value is mathematically correct, so de facto the period of discounting is of no relevance. However, due to the fact that these formulas bring \( NCF \) components to various moments of time, one ought to bear in mind that when comparing investment variants using this method it is recommended to use only one method of calculating NPV. Information regarding NPV value of a given investment variant, without the information concerning the type of used formula, can lead to selection of a non-optimal investment variant. As for IRR method, the period of discounting is of no relevance as IRR values will be equivalent regardless of selected moment of time. Nonetheless, the result will be different with the use of a modified version of IRR method. Using \( MIRR \) it is important to remember that selection
of the moment for which cash surplus will be discounted is of no relevance in only one case, i.e. when MIRR equals IRR. The applied conception of calculating MIRR ought to result from the assumed conception of calculating NPV, however, it is not common in practice. What is more, it ought to be stressed that for correct investment efficiency evaluation it is of key significance to include the total value of investment in the analysis along with its residual value. Nonetheless, it is alarming that most formulas provided in literature do not include this component. As a result, calculations produce a changeable value of investment efficiency indices depending on the length of discounting period. The shorter discounting period one assumes, the greater residual value is obtained.

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IMPLIKACJE SPOSOBU DYSKONTOWANIA I ZWIĄZKÓW MIĘDZY NPV, IRR I MIRR DLA OCENY EFEKTYWNOŚCI PROJEKTÓW INWESTYCYJNYCH

Ocena efektywności inwestycji projektów inwestycyjnych w gospodarce rynkowej opierać się powinna w głównej mierze na wykorzystaniu wskaźników opartych na technice dyskonta. Wskaźniki te, mimo iż znane są od wielu lat, w wielu przypadkach stosowane są niewłaściwie. Technika dyskonta wykorzystywana w tych wskaźnikach wymaga wyboru jednolitego momentu czasowego, na które są dyskontowane przepływy pieniężne jak również uwzględnienia wartości całej planowanej/realizowanej inwestycji wraz z jej wartością rezydualną. Wykorzystywanie tych wskaźników bez odpowiedniej wiedzy z zakresu nie tylko ekonomii ale i matematyki skutkować może niewłaściwą oceną efektywności analizowanych projektów inwestycyjnych, czego skutkiem może być podjęcie niewłaściwej decyzji i w konsekwencji narażenie firmy na straty. W artykule przedstawiono konsekwencje wyboru dyskontowania na różne momenty w czasie, dla dyskontowych wskaźników oceny efektywności inwestycji – NPV, IRR in MIRR. Zaprezentowano związki matematyczne zachodzące między tymi metodami oraz konsekwencje tych związków dla oceny opłacalności inwestycji. Wykazano, iż w przypadku metody IRR, nie ma znaczenia na jaki okres dyskontujemy, gdyż bez względu na wybrany moment czasowy wartości IRR będą zawsze sobie równe. Nie jest tak jednak przy wykorzystaniu zmodyfikowanej wersji metody IRR - MIRR. W tym przypadku, wybór momentu, na który zostaną zdyskontowane nadwyżki pieniężne nie ma znaczenia tylko w jednym przypadku, tj. gdy MIRR jest równa IRR. Wykorzystana koncepcja liczenia MIRR wynikać powinna z przyjętej koncepcji liczenia NPV, jednakże jak pokazuje praktyka w wielu przypadkach tak nie jest.

Słowa kluczowe: inwestycje, efektywność inwestycji, metody oceny efektywności inwestycji.

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