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ANALYSIS OF STEEL I-BEAM-COLUMNS CROSS-SECTION RESISTANCE WITH USE OF FINITE ELEMENT METHOD

An interaction of uniaxial bending and axial compression occurs very often in engineering practice and has been widely described in clause 6.2 of Eurocode 3. There are appropriate formulae given to verify the design cross-section resistance of steel structural members. Numerical aspects of constructing the cross-section resistance interaction curve of I-sections are presented in this paper through numerical modelling of the behaviour of stocky beam-columns with the relative slenderness ratio of 0.2 that distinguishes the range of characteristic values of the cross-section resistance and the member buckling resistance. Numerical study was carried out with respect to hot-rolled wide flange HEB/HEAA sections. The case of combined planar bending about one of the section principal axis (y or z) and axial compression was considered. The scope of numerical research includes different cross-section classes from class 1 to class 4. In case of cross-sections from class 1 to class 3, Geometrically and Materially Nonlinear Analysis (GMNA) is carried out. As far as class 4 cross-sections are concerned, Geometrically and Materially Nonlinear with Imperfections Analysis (GMNIA) is carried out in which geometrical imperfections due out-of-flatness deformations of section plate components are taken into account in the form of first local buckling mode. Plate section components are modelled using Finite Shell Elements (FSEs). FSE modelling technique using ABAQUS/Standard library is utilized. Numerical results in the form of interaction diagrams are compared with those of analytical ones presented in Eurocode 3. Differences occurring between the results from conducted FE analysis and those obtained with use of Eurocode's interaction formulae are discussed and factors causing these differences are outlined. Concluding remarks are drawn.

Keywords: cross-section resistance, compression and uniaxial bending, steel stocky I-beam-beams, FEM

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1. Introduction

Interaction of planar bending and compression occurs very often in engineering practice and has been widely described in PN-EN 1993, Part 1-1 [1]. In accordance with the above mentioned standard, in case of designing of steel members, the overall stability resistance (buckling resistance) and the cross-section resistance should be verified independently, the latter referred to the member most stressed section.

Additionally, in case of stocky elements that satisfy following conditions:

– members in compression:

$$\bar{\lambda} \leq 0.2 \text{ or } \frac{N_{Ed}}{N_{cr}} \leq 0.04 , \quad (1a)$$

– members in bending about the stronger axis of inertia:

$$\bar{\lambda}_{LT} \leq 0.2 \text{ or } \frac{M_{Ed}}{M_{cr}} \leq 0.04 , \quad (1b)$$

where: $\bar{\lambda}$, $\bar{\lambda}_{LT}$ – non-dimensional relative slendernesses with respect to compression and bending, respectively,

N_{cr} – elastic critical load with respect to given form of overall instability (flexural, torsional or torsional-flexural, respectively),

M_{cr} – elastic critical moment for lateral-torsional buckling,

the buckling effects may be ignored and only cross sectional checks apply.

In this paper, the issue of cross-section resistance is dealt with in reference to the resistance of stocky elements with doubly symmetrical I-shaped cross-section of the relative slenderness ratio of 0.2. In case like that, the member resistance may be verified on the basis of the criterion for the most critical cross-section resistance. In accordance with standard [1], regardless of the member relative slenderness ratio, three methods of proceeding while checking the cross-section resistance are actually used:

- for cross-sections belonging to classes from 1 to 3, application of **G**eometrically and **M**aterially **N**onlinear **A**nalysis of stocky elements with second order effects accounted for (GMNA according to [2]),
- for class 4 cross-sections, application of **G**eometrically and **M**aterially **N**onlinear with **I**mperfections **A**nalysis (GMNIA according to [2], taking into account geometric out-of-flatness imperfections associated with the first mode of slender web plate local buckling),
- application of the interaction formulae given in clause 6.2 of Eurocode 3, Part 1-1 [1] and complying with the requirements of Eurocode 3, Part 1-5 [3].

The main aim of this paper is to construct the cross-section resistance interaction curves of I-sections under compression and planar bending about principal axes (y,z) through numerical modelling of the behaviour of stocky beam-columns with the relative slenderness ratio of 0.2. The scope of numerical investigations includes different cross-section classes from class 1 to class 4. In case of cross-sections belonging to classes from 1 to 3, GMNA is carried out. As far as class 4 cross-sections are concerned, GMNIA is carried out in which slender plate geometrical imperfections are taken into account. Shell modelling technique using ABAQUS/Standard finite elements library is utilized [4, 5]. Numerical results in the form of interaction diagrams are compared with those of Eurocode 3, Part 1-1 [1]. Concluding remarks are drawn and directions of future research are outlined.

2. Numerical investigation program and FEM models of stocky elements

In numerical analysis with use of FEM, the resistance of stocky elements (identified with the cross-section resistance) was checked with respect to hot-rolled wide flange HEB/HEAA sections from class 1 to 4 cross-section. Effective lengths of members were determined as follows:

$$\bar{\lambda}_i = \sqrt{\frac{N_{pl,Rk}}{N_{cr,i}}} = 0.2 \text{, where } i = y,\ z. \tag{2}$$

Using the expression for the elastic critical load corresponding to flexural buckling, the following equation for the stocky column length limit is obtained:

$$l_{cr,i} = \sqrt{\frac{0.2^2 \cdot \pi^2 \cdot E \cdot I_i}{A \cdot f_y}}, \tag{3a}$$

and for the elastic critical load corresponding to torsional buckling:

$$l_{cr,T} = \sqrt{\frac{0.2^2 \cdot \pi^2 \cdot E \cdot I_\omega}{A \cdot f_y \cdot i_C^2 - 0.2^2 \cdot G \cdot I_T}}, \tag{3b}$$

where cross-section characteristics I_i, I_ω, I_T, A and i_C^2 (i_C is the radius of polar gyration and refer to [1] for other quantities notation) are referenced to the gross cross-section area, regardless of the cross-section class.

Classification of analyzed cross-sections and limiting lengths for stocky members are presented in Table 1. FE models are constructed with use of linear and rectangular shell elements that are available in the ABAQUS/Standard library [4, 5]. Approximate size of elements is in the range of 6 to 30 mm and depends upon the size of cross-section plate components of the member and the axis of bending (y or z). Boundary conditions and loads were applied in reference points RP_i (i=1,2), located at end sections and linked to edges of shell elements with use of *MPC* option available in ABAQUS/Standard [5].

FE models include simply supported beam-columns with effective lengths $l_{cr,i}$ according to Table 1 (where $i=y,z$). In order to ensure uniaxial bending (to avoid an influence of torsional instabilities) additional boundary conditions at web-flange junction were applied for all SFE nodes along the entire member length. Figure 1 presents the FEM model of an element in bending about stronger axis of inertia (y). In case of bending about weaker axis of inertia, boundary conditions were applied in an analogous way. To solve non-linear boundary value problems of buckling, ABAQUS program is used, in which the theory of moderately large deformation is implemented and available through an option NLGEOM [4, 5, 6]. The σ-ε characteristic of structural steel is described with use of an elastic-plastic model with isotropic hardening. In the elastic range, Young's modulus E equals to 210 N/mm^2 is taken into account. The Poisson's ratio is assumed to be equal to 0.3. In the plastic range, the minimum ductility assumptions of standard [1] are adopted, concerning the ultimate strength $f_u = 1.1 \cdot f_y$ and corresponding ultimate strain ε_u=0.15.

Table 1. List and classification of analyzed cross-sections

Tabela 1. Zestawienie i klasyfikacja analizowanych przekrojów

No.	Section	Class of cross-section		Stocky member flexural lengths $l_{cr,i}$ [mm]		Stocky member torsional length $l_{cr,T}$ [mm]
		Pure bending (about axis y)	Pure compression	Bending about axis y	Bending about axis z	
Hot-rolled sections						
1.	HEB 300	1	1	2444	1453	1393
2.	HE 550 AA	1	3	4083	1275	1431
3.	HE 650 AA	1	4	4758	1225	1424

When performing FE analysis, the conditions of constant values of the first order bending moment and compressive axial force along the entire length of I-section member is regarded. For this purpose, stress and displacement boundary conditions are applied at ends of analyzed elements in reference points RP_i. In the first step of analysis, bending moments is applied without any axial compression and then, the right section end displacement in the direction of member longitudinal axis proceeds. The maximum value of axial reaction for the left section end is determined that controls the utlimate strength of the end section in the presence of axial compression and uniaxial bending. Such a procedure is illustrated in Figure 2.

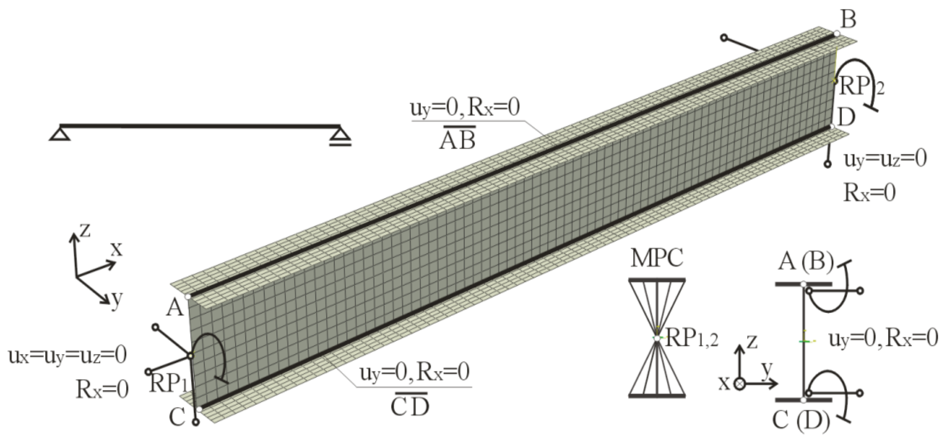

Fig. 1 FEM model of analyzed stocky members together with boundary conditions in case of bending about the stronger axis of inertia

Rys. 1 Model MES analizowanych elementów wraz z zadanymi warunkami brzegowymi w przypadku zginania względem osi większej bezwładności przekroju

The values of bending moments applied in reference end section points should be identified with the first order bending moments. Due to the application of the large deforamtion theory, the maximum value of bending moment is obtain in the middle of the span of analyzed element. Its amplified value is caused by the *P-δ* effect, resulting from an action of the axial force on the element deformed axis.

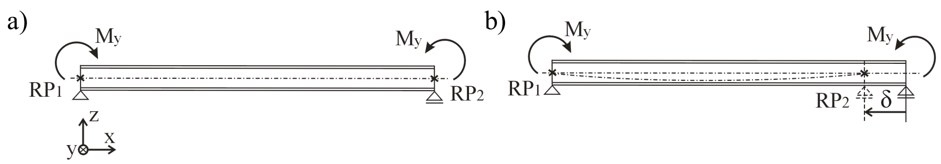

Fig. 2 FEM analysis procedure: a) STEP 1- applying bending moments in reference points RP_i, b) STEP 2 – member compression in the direction of longitudinal axis with the displacement control

Rys. 2 Przebieg analizy numerycznej: a) KROK 1 - przyłożenie momentów zginających w punktach odniesienia RP_i, b) KROK 2 – ściskanie elementu wzdłuż osi podłużnej pręta przy sterowaniu przemieszczeniowym

3. Resistance interaction curves of class 1 cross-section

Numerical calculations concern simply supported stocky elements of the HEB 300 cross-section. In case of the static scheme shown in Fig. 1, theoretical lengths of the member equal to those summarized in Table 1 ($l_{cr,i}$, where $i=y,z$) are taken under consideration hereafter. Design values of action effects $[N_{Ed}, M_{Ed}]$ used to construct the resistance interaction curves are those obtained from FE analysis in the appropriate reference point RP_i. In case of bending about the stronger axis of inertia (y), numerical results of GMNA are marked by A in Fig. 3, connected by a dotted line representing the interaction diagram. It coincides quite well with that obtained by using Eurocode's formulae (solid line marked by EC in Fig. 3). A slight difference in results could be seen only for high values of $M_{y,Ed}/M_{pl,Rk}$ ratio.

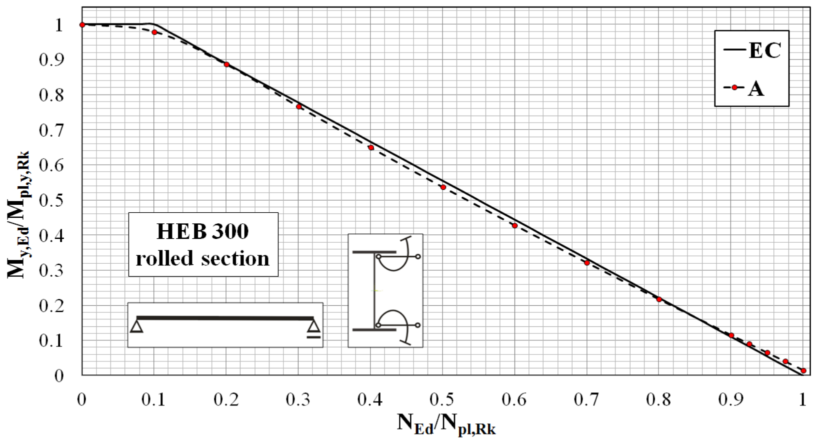

Fig. 3 The resistance interaction curve of HEB 300 cross-section under compression and uniaxial bending about the stronger axis of inertia

Rys. 3 Interakcyjna krzywa nośności przekroju HEB 300 ściskanego i zginanego względem osi większej bezwładności przekroju

Axial compression results in association of bending moments about weaker axis of inertia (z) are given in Fig. 4. In case of bending about weaker axis of inertia (z), noteworthy is the fact of reaching a non-zero value of bending moment $M_{z,Ed}$ for maximum value of an axial force N_{Ed} (point $N_{Ed}/N_{pl,Rk}$=1 on the horizontal axis in Fig. 4). This is due to the plastic reserve of class 1 cross-sections resulting from the consideration of strain hardening effect of steel (note that the structural steel σ-ε characteristic exhibits the isotropic hardening in the plastic range). The resistance interaction curves of doubly symmetrical I-shaped cross-section, under compression and planar bending about weaker axis of inertia (z), have a shape that is different from that corresponding to stronger axis bending. This results from the fact that bending about z axis causes yielding of two flanges of a rectangular shape so that the interaction curve is close to that obtained in case of a rectangular cross-section.

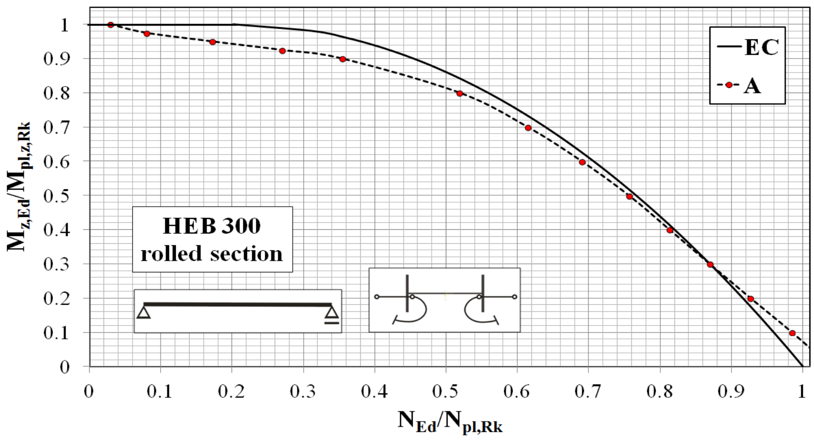

Fig. 4 The resistance interaction curve of HEB 300 cross-section under compression and uniaxial bending about the weaker axis of inertia

Rys. 4 Interakcyjna krzywa nośności przekroju HEB 300 ściskanego i zginanego względem osi mniejszej bezwładności przekroju

4. Resistance interaction curves of class 3 cross-section

The resistance interaction curves were determined with regard to HE 550 AA cross-section, which belongs to class 3 with respect to pure compression and class 1 in case of bending about stronger axis of inertia (*y*). In this analysis, the simply supported boundary conditions are taken into consideration. GMNA is carried out. As it has been assumed earlier, design values of action effects $[N_{Ed}, M_{Ed}]$ to construct the resistance interaction curves are taken in the appropriate reference point RP_i. Fig. 5 presents the results for compression and bending about section stronger axis of inertia (*y*). Although numerical results are in this case above those based on Eurocode's formulae, they coincide quite well with those obtained from Eurocode's formulae. In case of bending about weaker axis of inertia (*z*), see Fig. 6, obtained interaction curves are completely different from those corresponding to Eurocode 3. For bending about the axis *z*, the web practically does not influence the design bending resistance. In accordance with code [1], the linear interaction equation is very conservative since it does not take into account the plastic reserve of class 1 flanges in bending about axis (*z*). Dotted line in Fig. 6 presents the plastic interaction curve (*EC_p* curve). One may note that up to ¾ of the bending capacity $M_{pl,z,Rd}$, the interaction curve is close to that of the plastic section. An approximate interaction curve in Fig. 6 may be suggested as a bilinear.

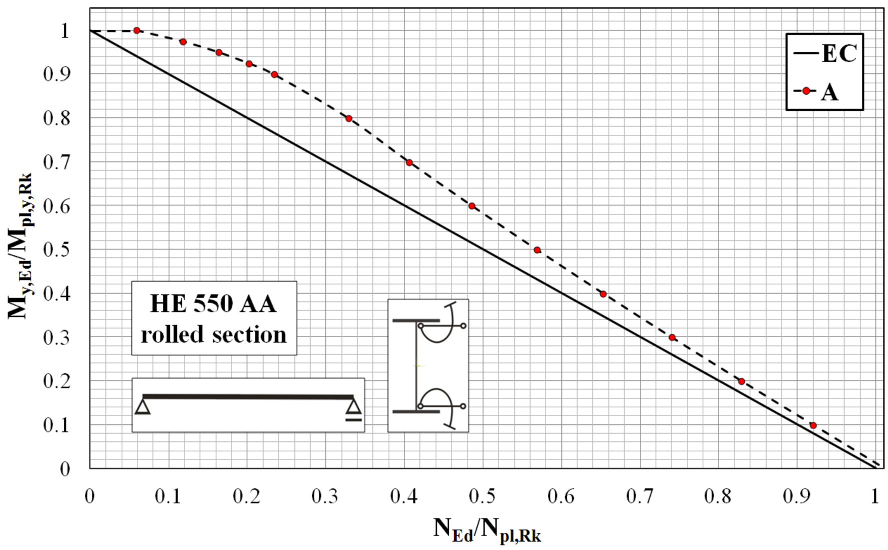

Fig. 5 The resistance interaction curve of HE 550 AA cross-section under compression and uniaxial bending about the stronger axis of inertia

Rys. 5 Interakcyjna krzywa nośności przekroju HE 550 AA ściskanego i zginanego względem osi większej bezwładności przekroju

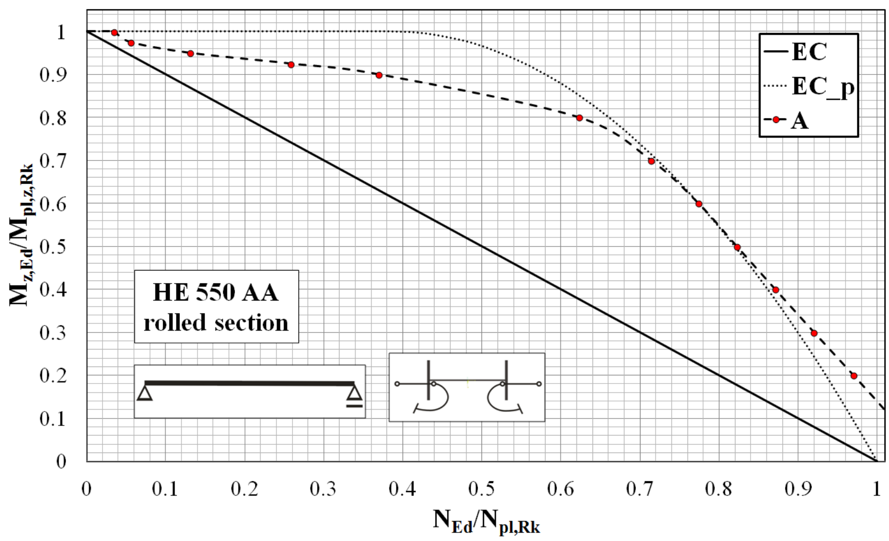

Fig. 6 The resistance interaction curve of HE 550 AA cross-section under compression and uniaxial bending about the weaker axis of inertia

Rys. 6 Interakcyjna krzywa nośności przekroju HE 550 AA ściskanego i zginanego względem osi mniejszej bezwładności przekroju

5. Resistance interaction curves of class 4 cross-section

The resistance interaction curves were determined with regard to HE 650 AA cross-section, which belongs to cross-section class 4 with respect to pure compression and class 1 in bending about stronger axis of inertia (y). The same boundary conditions and other assumptions with regard to action effects $[N_{Ed}, M_{Ed}]$ used to construct the resistance interaction curves described in the previous parts of this paper are taken into consideration.

The resistance of cross-sections may be determined using the effective area A_{eff} of thin web in compression for class 4 sections [3]. Effective cross-section area should be determined on the basis of linear strain distributions with the attainment of yield strain in the mid-plane of the compression plate.

In order to analyze the class 4 cross-section, local buckling of thin web plate is taken under consideration. Thus, local geometrical imperfections with reference to thin web out-of-flatness are taken into account in numerical analysis of stocky element considered hereafter. The shape of local geometrical imperfections corresponds to the first web local buckling mode according to LBA. Buckling analysis was carried out with respect to pure compression. Fig. 7 illustrates the first web buckling mode with respect to elastic plate instability in case of considered HE 650 AA cross-section. The geometry of imperfect element in the initial configuration was adopted as the scaled first plate buckling mode of the web. The value of the scaling factor corresponds to the maximum imperfection provided in the standard [3]. The magnitude of local imperfection amplitude is assumed to be of 1/200 of smaller dimension of the web panel. As a simplification, the out-of-flatness web geometrical imperfections corresponding to pure compression are adopted for determining the resistance interaction curve in the entire range of dimensional action effects $N_{Ed}/N_{pl,Rk}$ and $M_{Ed}/M_{pl,Rk}$.

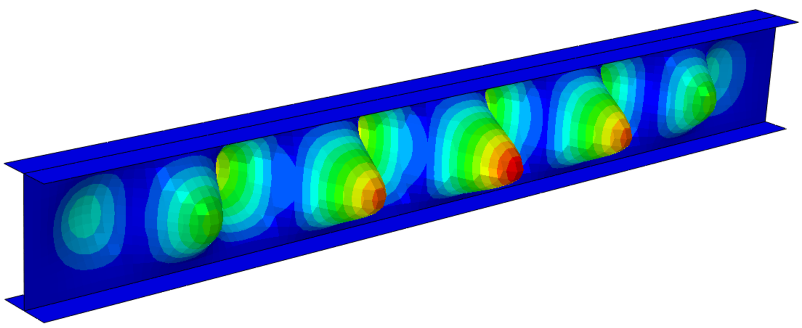

Fig. 7 The first buckling mode corresponding to elastic buckling of web plate in case of HE 650 AA cross-section

Rys. 7 Pierwsza postać funkcji przemieszczeń odpowiadająca sprężystemu wyboczeniu płytowemu środnika w przypadku elementu o przekroju HE 650 AA

In order to investigate the influence of imperfections on the resistance of elements GMNIA is carried out. For comparisons, two tasks are solved in order to construct interaction curves for ideal geometry and initially deformed geometry. In case of bending about the stronger axis of inertia (*y*), numerical results coincide quite well with those obtained from Eurocode's formulae (see Fig. 8). The influence of initial imperfections on the resistance of analyzed member is significant when the value of $N_{Ed}/N_{pl,Rk}$ ratio is high and it is less important for high values of $M_{y,Ed}/M_{pl,y,Rk}$. In case of pure compression ($N_{Ed}/N_{pl,Rk}$=1, $M_{y,Ed}/M_{pl,y,Rk}$=0) the results of GMNIA and Eurocode are coinciding.

Fig. 9 shows the results corresponding to compression and bending about weaker axis (*z*). In case of bending about weaker axis of inertia, obtained interaction curves are convex, whereas those of Eurocode 3 are linear. Dotted line represents a semi-plastic interaction curve (*EC_p* curve) in which the section compression resistance is modified and based on the effective area of thin web plate in compression (i.e. A_{eff} as for class 4 sections) instead of the gross area as for class 1 sections. Comparing the obtained interaction curves and the semi-plastic interaction one, similar conclusions hold as those stated for bending about stronger axis. Effect associated with the inclusion of imperfections is similar to that discussed earlier in case of bending about stronger axis of inertia (*y*).

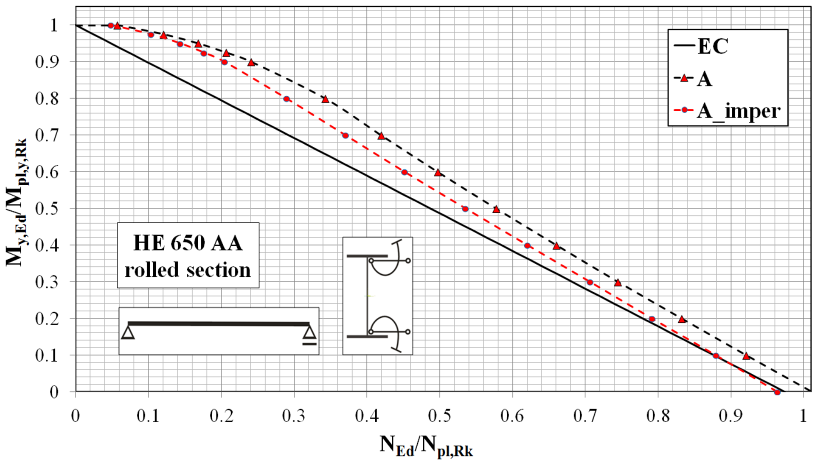

Fig. 8 The resistance interaction curve of HE 650 AA cross-section under compression and uniaxial bending about the stronger axis of inertia; results refer to an influence of the local buckling of the thin web plate of class 4 on the stocky member resistance

Rys. 8 Interakcyjna krzywa nośności przekroju HE 650 AA ściskanego i zginanego względem osi większej bezwładności przekroju; wyniki dotyczą wpływu niestateczności miejscowej smukłego środnika klasy 4 przy ściskaniu na nośność pręta krępego

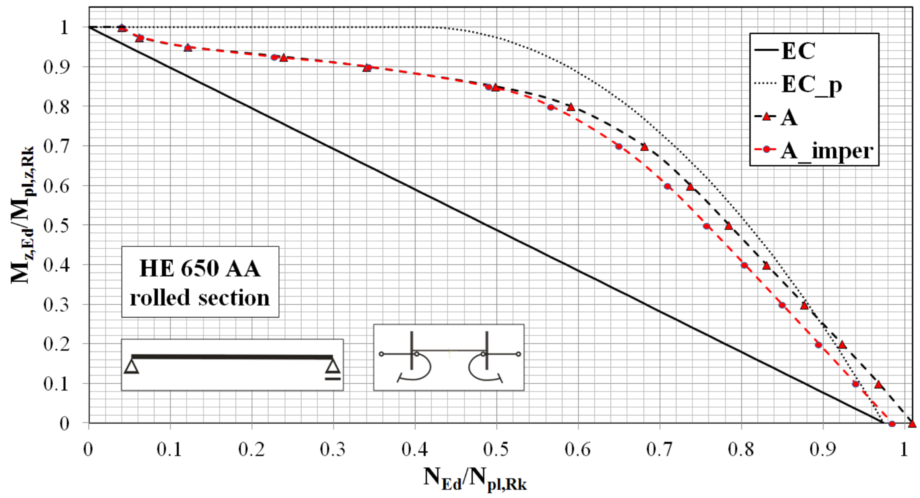

Fig. 9 The resistance interaction curve of HE 650 AA cross-section under compression and uniaxial bending about the weaker axis of inertia; results refer to an influence of the local buckling of the thin web plate of class 4 on the stocky member resistance

Rys. 9 Interakcyjna krzywa nośności przekroju HE 650 AA ściskanego i zginanego względem osi mniejszej bezwładności przekroju; wyniki dotyczą wpływu niestateczności miejscowej smukłego środnika klasy 4 przy ściskaniu na nośność pręta krępego

The deformed member profile of HE 650 AA cross-section with imperfections in pure compression is shown in Fig. 10. It corresponds to the limit point on the equilibrium path at which the section resistance is reached.

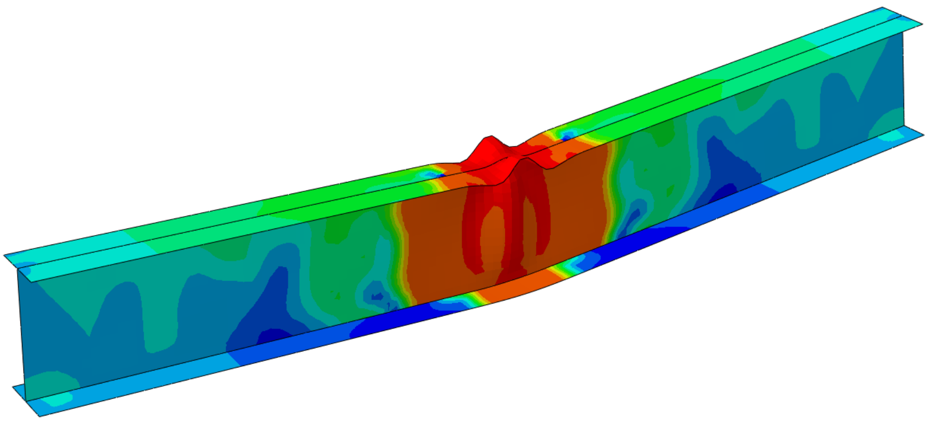

Fig. 10 Deformed member of HE 650 AA cross-section with local imperfections after exceeding the resistance in case of pure compression

Rys. 10 Zdeformowany pręt o przekroju HE 650 AA z zaimplementowanymi imperfekcjami po przekroczeniu nośności w przypadku czystego ściskania

6. Concluding remarks

Evaluation of the cross section resistance under combined axial compression and planar bending moment in respect to principal axes (*y*,*z*) is dealt with in this paper. The cross section resistance interaction curves are developed on the basis of second-order analysis GMNA for stocky members of class 1-3 sections and GMNIA for stocky elements of class 4. In order to analyze the class 4 cross-section, geometrical imperfections are taken into account as corresponding to the first web plate buckling mode with amplitude scaled according to the recommendations of [3].

In case of class 1 and 2 sections, obtained interaction curves are in a good agreement with those based on Eurocode 3 formulae. Less accurate results are obtained for elements under compression and bending about the weaker axis of inertia (*z*). Furthermore, noteworthy is the fact of reaching a non-zero value of bending moment $M_{z,Ed}$ for the maximum value of axial force N_{Ed} (for $N_{Ed}/N_{pl,Rk}$=1). This is due to the plastic reserve of class 1 and 2 cross-sections when the structural steel σ-ε characteristic is modelled as an elastic-plastic with isotropic strain-hardening in the post-elastic range.

In case of members belonging to classes 3 and 4 with respect to pure compression and class 4 with regard to bending about stronger axis (*y*), obtained interaction curves are closer to linear ones in case of bending about stronger axis of inertia (*y*). In case of bending about weaker axis of inertia (*z*), interaction curves are convex, whereas those of Eurocode 3 are linear. This indicates that there is a room for further improvement of codification rules.

The paper presents the specific case of stocky members under compression and bending about one of principal axes (*y, z*). Future research should take into consideration an influence of the member slenderness on the buckling resistance in case of in-plane bending and compression (no lateral-torsional buckling), assuming various cross-section classes. Future investigations should also account for the effect of lateral-torsional instability.

Bibliography

[1] Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings.

[2] Eurocode 3: Design of steel structures – Part 1-7: Strength and stability of planar plated structures subject to out of plane loading.

[3] Eurocode 3: Design of steel structures – Part 1-5: Plated structural elements.

[4] ABAQUS *Theory manual*, Version 6.1., Hibbitt, Karlsson and Sorensen, Inc., Pawtucket, 2000.

[5] ABAQUS/Standard *User's manual*, Version 6.1., Hibbitt, Karlsson and Sorensen, Inc., Pawtucket, 2000.

[6] Jemioło S., Gajewski M.: Hiper-elasto-plasticity. Seria wydawnicza Monografie Zakładu Wytrzymałości Materiałów, Teorii Sprężystości i Plastyczności, Tom III, OWPW, 2014 (in Polish).

ANALIZA NOŚNOŚCI STALOWYCH PRZEKROJÓW DWUTEOWYCH ŚCISKANYCH I ZGINANYCH W JEDNEJ PŁASZCZYŹNIE PRZY ZASTOSOWANIU MES

S t r e s z c z e n i e

Zagadnienie jednokierunkowego zginania oraz ściskania jest spotykane w praktyce inżynierskiej bardzo często i zostało szeroko opisane w pkt. 6.2 normy PN-EN 1993-1-1. Podano w niej odpowiednie formuły do sprawdzania nośności prętowych konstrukcji metalowych. W niniejszej pracy skupiono się na numerycznych aspektach oceny nośności stalowych przekrojów dwuteowych rozumianej jako nośność pręta krępego poddanego jednocześnie ściskaniu oraz jednokierunkowemu zginaniu. Studium numeryczne wykonano w odniesieniu do kształtowników walcowanych na gorąco z grupy szerokostopowych HEB/HEAA, zginanych względem głównych osi bezwładności przekroju (y, z). Zakres badań obejmuje przekroje zaliczane do klas od 1 do 4. W przypadku prętów krępych o przekroju należącym do klas od 1 do 3 wykonano pełną geometrycznie i materiałowo nieliniową analizę z uwzględnieniem efektów II rzędu (GMNA). W przypadku prętów o przekroju klasy 4 zastosowano zaś analizę GMNIA, uwzględniającą dodatkowo imperfekcje geometryczne ścianek smukłych. Modele numeryczne prętów wykonano z zastosowaniem powłokowych elementów skończonych. Do wykonania analiz numerycznych wykorzystano program komputerowy ABAQUS/Standard. Uzyskane wyniki krzywych granicznych nośności przekroju porównano z wynikami uzyskanymi z zastosowaniem formuł normowych. Sformułowano wnioski oraz kierunki dalszych badań.

Słowa kluczowe: nośność przekroju, dwuteowniki walcowane na gorąco, ściskanie i zginanie jednokierunkowe, metoda elementów skończonych MES

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